

# thm\_2Earithmetic\_2Enum\_\_case\_\_compute (TMX-uGCdRRNJs38QGkRYyyBmRHEsgQxikcYp)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2Enum\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_{27a}. nonempty\ A_{27a} \Rightarrow c\_2Earithmetic\_2Enum\_CASE\ A_{27a} \in (((A_{27a})^{(A_{27a}^{ty\_2Enum\_2Enum})})^{A_{27a}})^{ty\_2Enum\_2Enum} \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (ap (ap (c\_2Emin\_2E\_3D (2^{A_{27a}}) (V0P))) (V0P)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2. (inj\_o (t1 = t2))))))$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p)) \text{ else } (\lambda x. x \in A \wedge \neg p))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A_{27a} : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_{27a}. (\lambda V2t2 \in A_{27a}. (ap (c\_2Ebool\_2E\_21 2) (inj\_o (t1 = t2))))))$

**Definition 9** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2. (ap (ap (c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2EF)))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (3)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (4)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (5)$$

**Definition 10** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$   
Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (6)$$

**Definition 11** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP).$

**Definition 12** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Enum\_2E0) V0v) V1f) = V0v)) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3v \in A\_27a.(\forall V4f \in (A\_27a^{ty\_2Enum\_2Enum}).((ap (ap (c\_2Earithmetric\_2Enum\_CASE A\_27a) (ap c\_2Enum\_2ESUC V2n)) V3v) V4f) = (ap V4f V2n))))))$   
Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty A\_27a \Rightarrow & ((\forall V0v \in A\_27a.(\forall V1f \in (A\_27a^{ty\_2Enum\_2Enum}).((ap (ap (ap (c\_2Earithmetric\_2Enum\_CASE A\_27a) c\_2Enum\_2E0) V0v) V1f) = V0v)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. \\ & (\forall V3v \in A\_27a.(\forall V4f \in (A\_27a^{ty\_2Enum\_2Enum}).((ap (ap (c\_2Earithmetric\_2Enum\_CASE A\_27a) (ap c\_2Enum\_2ESUC V2n)) V3v) V4f) = (ap V4f V2n))))))) \end{aligned} \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (10)$$

Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty A\_27a \Rightarrow & (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V0t1) V1t2) = V1t2)))) \end{aligned} \quad (12)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap c\_2Enum\_2ESUC V0n) = c\_2Enum\_2E0))) \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p (ap V0P c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum.((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC V1n))))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum.(p (ap V0P V2n)))))) \end{aligned} \quad (14)$$

Assume the following.

$$(((ap\ c\_2Eprim\_rec\_2EPRE\ c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (\forall V0m \in ty\_2Enum\_2Enum. ((ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Enum\_2ESUC\ V0m)) = V0m))) \quad (15)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0f \in A\_27a. (\forall V1g \in \\ & (A\_27a^{ty\_2Enum\_2Enum}). (\forall V2n \in ty\_2Enum\_2Enum. ((ap\ (ap \\ & (ap\ (c\_2Earithmetic\_2Enum\_CASE\ A\_27a)\ V2n)\ V0f)\ V1g) = (ap\ (ap \\ & (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Enum\_2Enum) \\ & V2n)\ c\_2Enum\_2E0))\ V0f)\ (ap\ V1g\ (ap\ c\_2Eprim\_rec\_2EPRE\ V2n))))))) \end{aligned}$$