

thm\_2Earithmetic\_2Enum\_\_case\_eq  
 (TMbPXjt6q24yRso6LXwQp659P4XkqHXAmrG)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let  $c\_2Earithmetic\_2Enum\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow c_2Earithmetic_2Enum_CASE\ A_27a \in (((A_27a^{(A_27a^{ty\_2Enum\_2Enum})})^{A_27a})^{ty\_2Enum\_2Enum}) \quad (2)$$

**Definition 1** We define  $c_2Emin_2E_3D$  to be  $\lambda A. \lambda x \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c_2Ebool_2ET$  to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c_2Ebool_2E_21$  to be  $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (V0P)))$

**Definition 4** We define  $c_2Ebool_2EF$  to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c_2Emin_2E_3D_3D_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o (p \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c_2Ebool_2E_7E$  to be  $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c_2Enum_2EZERO\_REP \in omega \quad (3)$$

Let  $c\_2Enum\_2EAABS\_num : \iota$  be given. Assume the following.

$$c_2Enum_2EAABS\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

**Definition 7** We define  $c_2Enum_2E0$  to be  $(ap c_2Enum_2EAABS\_num c_2Enum_2EZERO\_REP)$ .

**Definition 8** We define  $c_2Ebool_2E_5C_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj\_o (t1 = t2))))$

**Definition 9** We define  $c_2Ebool_2E_2F_5C$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj\_o (t1 = t2))))$

**Definition 10** We define  $c\_2Emin\_2E40$  to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge P x) \text{ else } \perp$

**Definition 11** We define  $c\_2Ebool\_2E3F$  to be  $\lambda A. \lambda a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c\_2Emin\_2E40 a) V0)))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (6)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num m)$

Assume the following.

$$\begin{aligned} \forall A. \exists a. \text{nonempty } A \Rightarrow & ((\forall V0v \in A. \forall V1f \in \\ & (A \cdot 27a^{ty\_2Enum\_2Enum}). ((ap (ap (ap (c\_2Earithmetic\_2Enum\_CASE \\ & A \cdot 27a) c\_2Enum\_2E0) V0v) V1f) = V0v))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. \\ & (\forall V3v \in A \cdot 27a. (\forall V4f \in (A \cdot 27a^{ty\_2Enum\_2Enum}). ((ap \\ & (ap (ap (c\_2Earithmetic\_2Enum\_CASE A \cdot 27a) (ap c\_2Enum\_2ESUC \\ & V2n)) V3v) V4f) = (ap V4f V2n))))))) \end{aligned} \quad (7)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg(c\_2Enum\_2E0 = (ap c\_2Enum\_2ESUC V0n)))) \quad (8)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((V0m = c\_2Enum\_2E0) \vee (\exists V1n \in ty\_2Enum\_2Enum. (V0m = (ap c\_2Enum\_2ESUC V1n))))) \quad (9)$$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A. \exists a. \text{nonempty } A \Rightarrow & ((\forall V0t \in 2. ((\exists V1x \in \\ & A \cdot 27a. (p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (14)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (15)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (16)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (17)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1a \in A\_27a.((\exists V2x \in A\_27a.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a))))) \quad (18)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0m) = (ap c\_2Enum\_2ESUC V1n)) \Leftrightarrow (V0m = V1n))) \quad (19)$$

### Theorem 1

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1zc \in A\_27a.(\forall V2sc \in (A\_27a^{ty\_2Enum\_2Enum}).(\forall V3v \in A\_27a.(((ap (ap (ap (c\_2Earithmetic\_2Enum\_CASE A\_27a) V0n) V1zc) V2sc) = V3v) \Leftrightarrow (((V0n = c\_2Enum\_2E0) \wedge (V1zc = V3v)) \vee (\exists V4x \in ty\_2Enum\_2Enum.((V0n = (ap c\_2Enum\_2ESUC V4x)) \wedge ((ap V2sc V4x) = V3v))))))))))) \end{aligned}$$