

thm_2Earithmetic_2Etransitive__LESS
 (TMUbkMix-
 Agx3cDLjrGGMPB5vTSQq5r7MoSF)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Definition 9 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 10 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40$

Definition 11 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in \text{ty_2Enum_2Enum}. \lambda V1n \in \text{ty_2Enum_2Enum}$

Definition 12 We define `c_2Erelation_2ETC` to be $\lambda A_{27a} : \iota. \lambda V0R \in ((2^{A_{27a}})^{A_{27a}}). \lambda V1a \in A_{27a}. \lambda V2b$

Definition 13 We define `c_2Erelation_2Etransitive` to be $\lambda A_{27a} : \iota. \lambda V0R \in ((2^{A_{27a}})^{A_{27a}}). (\text{ap (c_2Ebool_2E_3F$

Assume the following.

$$\text{True} \tag{5}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((\text{True} \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ & (p \ V0t)) \wedge (((\text{False} \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow \text{False}) \Leftrightarrow \neg(\\ & p \ V0t)))))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\text{c_2Eprim_rec_2E_3C} = (\text{ap (c_2Erelation_2ETC ty_2Enum_2Enum)} \\ & (\lambda V0x \in \text{ty_2Enum_2Enum}. (\lambda V1y \in \text{ty_2Enum_2Enum}. (\text{ap (ap (} \\ & \text{c_2Emin_2E_3D ty_2Enum_2Enum) V1y) (ap c_2Enum_2ESUC V0x)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0R \in ((2^{A_{27a}})^{A_{27a}}). \\ & (p \ (\text{ap (c_2Erelation_2Etransitive } A_{27a}) \ (\text{ap (c_2Erelation_2ETC} \\ & A_{27a}) \ V0R)))) \end{aligned} \tag{8}$$

Theorem 1

$$(p \ (\text{ap (c_2Erelation_2Etransitive ty_2Enum_2Enum) c_2Eprim_rec_2E_3C}))$$