

thm_2Ebag_2EBAG_-CARD_-THM (TMFrByGo-jZbPymrVyVWcJrBbCRpNHdzLvqK)

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Let $c_2Enum_2ZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2ABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2ABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be ($ap\ c_2Enum_2ABS_num\ c_2Enum_2ZERO_REP$).

Definition 3 We define $c_2Earithmetic_2ZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2SUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2SUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 4 We define c_2Ebool_2ET to be ($ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$)

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2ABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 n) V0)$

Definition 8 We define $c_2Earthmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t\in 2.V0t))$.

Definition 10 We define $c_{\text{2Emin_3D_3D_3E}}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_{\text{c_Ebool_2E_2F_5C}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_{\text{c_Ebool_2E_21}}) 2)) (\lambda V2t \in$

Definition 12 We define $c_{\text{2Emin_2E_40}}$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \text{ } x)) \text{ then } (\lambda x. x \in A \wedge \text{of type } \iota \Rightarrow \iota)$.

Definition 14 We define $c_2EBag_2EBAG_INSERT$ to be $\lambda A.27a : \iota. \lambda V0e \in A.27a. \lambda V1b \in (ty_2Enum_2E$

Definition 15 We define $c_2EBag_2EBAG_DELETE$ to be $\lambda A._27a : \iota.\lambda V0b0 \in (ty_2Enum_2Enum^A_{27a}).\lambda$

Definition 16 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E))$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^A_{27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2\text{Eprim_rec_E_3C}$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 19 We define c_2 Earthmetic_2E_3E to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 20 We define $c_{\text{CBool}} : \mathbf{CBool} \rightarrow \mathbf{CBool}$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_{\text{CBool}}) 2E_21 2)) (\lambda V2t \in$

Definition 21 We define c_2Earthmetic_2E_3E_3D to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 22 We define $c \in \text{bag_BAG_INN}$ to be $\lambda A \lambda 27a : \lambda V 0e \in A \lambda 27a. \lambda V 1n \in \text{ty_Enum_Enum}$

Definition 23 We define $c_2EBAG_2EBAG_IN$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1b \in (ty_2Enum_2Enum'$

Definition 24 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 25 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota.(ap(c_2Ecombin_2EK\ ty_2Enum_2En$

Let $c_2EBag_2EBAG_CARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow c_2Eb_{\text{ag_2EBAG_CARD}} A_27a \in (\text{ty_2Enum_2Enum}^{(\text{ty_2Enum_2Enum}', \dots)}, \dots) \quad (7)$$

Definition 26 We define $c \in \text{EBAG_EBAG_CARD_RELn}$ to be $\lambda A.27a : \iota.\lambda V0b \in (\text{ty_Enum_Enum}^A)^{27}$

Definition 27 We define $c_2EBag_2EFINITE_BAG$ to be $\lambda A.\lambda 27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^A)^{27a}.$ (ap

Definition 28 We define $c_{\text{Ecombin_2ES}}$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.(\lambda V0f \in ((A_\!27c^{A_\!27b})^A)^{A_\!27a})$

Definition 29 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota. (ap (ap (c_2Ecombin_2ES A_27a) (A_27a^{A_27a})) A_27a)$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (9)$$

Definition 30 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2B$

Let $c_2Earithmetic_2EEEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (12)$$

Definition 31 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap c_2Enum_2ESUC (ap$

Definition 32 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 33 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic$

Definition 34 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\ & (13) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \\ & (14) \end{aligned}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
 & (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p)))))) \\
 \end{aligned} \tag{15}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \\
 c_2Enum_2E0) V0n))) \tag{16}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0m) = (ap (ap \\
 & c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
 & c_2Earithmetic_2EZERO)))))) \\
 \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
 & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
 & (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
 & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m) V1n) = (ap \\
 & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
 & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
 & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
 & V0m) V1n))))))) \\
 \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & ((ap (ap c_2Earithmetic_2E_2B V0m) V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = \\
 & c_2Enum_2E0) \wedge (V1n = c_2Enum_2E0)))))) \\
 \end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2B V0m) \\
 & V2p) = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) \Leftrightarrow (V0m = V1n)))))) \\
 \end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
 & \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
 & V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
 & ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p))))))) \\
 \end{aligned} \tag{21}$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) V1c)) V1c) = V0a))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V1n)) V0m))))))) \quad (25)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)) V0n)))) \quad (26)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D V1a) V2b))) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1e1 \in A_27a. (\forall V2e2 \in A_27a. ((p (ap (ap (c_2Ebag_2EBAG_IN A_27a) V1e1) (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) V2e2) V0b)))) \Leftrightarrow \\ & ((V1e1 = V2e2) \vee (p (ap (ap (c_2Ebag_2EBAG_IN A_27a) V1e1) V0b))))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0x \in A_{27a}.(\forall V1b \in \\ (ty_2Enum_2Enum^{A_{27a}}).(\neg((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a}) \\ V0x)\ V1b) = (c_2Ebag_2EMPTY_BAG\ A_{27a})))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b1 \in (ty_2Enum_2Enum^{A_{27a}}). \\ (\forall V1b2 \in (ty_2Enum_2Enum^{A_{27a}}).(\forall V2x \in A_{27a}.((\\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2x)\ V0b1) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V2x)\ V1b2)) \Leftrightarrow (V0b1 = V1b2)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b \in (ty_2Enum_2Enum^{A_{27a}}). \\ (\forall V1e1 \in A_{27a}.(\forall V2e2 \in A_{27a}.((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V1e1)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2e2)\ V0b)) = \\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2e2)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V1e1)\ V0b))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b \in (ty_2Enum_2Enum^{A_{27a}}). \\ (\forall V1e \in A_{27a}.((p\ (ap\ (ap\ (c_2Ebag_2EBAG_IN\ A_{27a})\ V1e) \\ V0b)) \Rightarrow (\exists V2b_{27} \in (ty_2Enum_2Enum^{A_{27a}}).(p\ (ap\ (ap\ (ap\ (\\ c_2Ebag_2EBAG_DELETE\ A_{27a})\ V0b)\ V1e)\ V2b_{27}))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0a \in A_{27a}.(\forall V1M \in \\ (ty_2Enum_2Enum^{A_{27a}}).(\forall V2b \in A_{27a}.(\forall V3N \in (ty_2Enum_2Enum^{A_{27a}}). \\ (((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V0a)\ V1M) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V2b)\ V3N)) \Leftrightarrow ((V1M = V3N) \wedge (V0a = V2b)) \vee (\exists V4k \in (ty_2Enum_2Enum^{A_{27a}}). \\ ((V1M = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2b)\ V4k)) \wedge (V3N = \\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V0a)\ V4k))))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_{27a}) \\ (c_2Ebag_2EMPTY_BAG\ A_{27a}))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b \in (ty_2Enum_2Enum^{A_{27a}}). \\ ((p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_{27a})\ V0b)) \Rightarrow (\forall V1e \in A_{27a}. \\ (p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_{27a})\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V1e)\ V0b))))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{(ty_2Enum_2Enum^{A_{27a}})})) \\
& (((p (ap V0P (c_2Ebag_2EMPTY_BAG A_{27a}))) \wedge (\forall V1b \in (ty_2Enum_2Enum^{A_{27a}}))) \\
& \quad (((p (ap (c_2Ebag_2FINITE_BAG A_{27a}) V1b)) \wedge (p (ap V0P V1b))) \Rightarrow \\
& \quad (\forall V2e \in A_{27a}. (p (ap V0P (ap (ap (c_2Ebag_2EBAG_INSERT A_{27a}) \\
& \quad V2e) V1b)))))) \Rightarrow (\forall V3b \in (ty_2Enum_2Enum^{A_{27a}}). ((p (ap \\
& \quad (c_2Ebag_2FINITE_BAG A_{27a}) V3b)) \Rightarrow (p (ap V0P V3b)))))) \\
& \tag{36}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_{27a}}). \\
& ((p (ap (c_2Ebag_2FINITE_BAG A_{27a}) V0b)) \Rightarrow (p (ap (ap (c_2Ebag_2EBAG_CARD_RELN \\
& \quad A_{27a}) V0b) (ap (c_2Ebag_2EBAG_CARD A_{27a}) V0b)))))) \\
& \tag{37}
\end{aligned}$$

Assume the following.

$$True \tag{38}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{40}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{41}$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{27a}. (p V0t) \Leftrightarrow (p V0t))) \tag{42}$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0t \in 2. ((\exists V1x \in A_{27a}. (p V0t) \Leftrightarrow (p V0t))) \tag{43}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \tag{44}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{45}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ & (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (49)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \quad (50)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (51)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\ True)) \quad (52)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\ & A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & (p V0t)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in \\ & A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p (ap V0P V2x))))))) \end{aligned} \quad (55)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}})). ((\neg(\exists V1x \in A_{\text{27a}}.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_{\text{27a}}.(\neg(p (ap V0P V2x))))) \quad (56)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{\text{27a}}})). (((p V0P) \wedge (\forall V2x \in A_{\text{27a}}.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_{\text{27a}}.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (57)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}})). (\forall V1Q \in 2. (((\exists V2x \in A_{\text{27a}}.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A_{\text{27a}}.((p (ap V0P V3x)) \vee (p V1Q))))) \quad (58)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{\text{27a}}})). (((p V0P) \vee (\exists V2x \in A_{\text{27a}}.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A_{\text{27a}}.((p V0P) \vee (p (ap V1Q V3x))))) \quad (59)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}})). (\forall V1Q \in 2. (((\exists V2x \in A_{\text{27a}}.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in A_{\text{27a}}.(p (ap V0P V3x)) \wedge (p V1Q))))) \quad (60)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{\text{27a}}})). (((\exists V2x \in A_{\text{27a}}.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \wedge (\exists V3x \in A_{\text{27a}}.(p (ap V1Q V3x))))) \quad (61)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_{\text{27a}}})). (((\forall V2x \in A_{\text{27a}}.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_{\text{27a}}.((p (ap V1P V3x)) \vee (p V0Q)))) \quad (62)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{\text{27a}}})). (((\forall V2x \in A_{\text{27a}}.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A_{\text{27a}}.(p (ap V1Q V3x))))) \quad (63)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{\text{27a}}})). (((\forall V2x \in A_{\text{27a}}.((p V0P) \Rightarrow (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \Rightarrow (\forall V3x \in A_{\text{27a}}.(p (ap V1Q V3x))))) \quad (64)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (67)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (p V1B))))))) \quad (68)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \wedge ((p V1B) \wedge (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C)))))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A)))))) \quad (70)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (71)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (72)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (73)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1))))) \quad (74)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))))) \quad (75)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0P \in (2^{A_{\text{27a}}})). (\forall V1a \in \\ A_{\text{27a}}. ((\exists V2x \in A_{\text{27a}}. ((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (\\ ap V0P V1a)))) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & (\forall V0f \in (2^{A_{\text{27a}}})). (\forall V1v \in \\ A_{\text{27a}}. ((\forall V2x \in A_{\text{27a}}. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (\\ ap V0f V1v)))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & \forall A_{\text{27b}}. \text{nonempty } A_{\text{27b}} \Rightarrow \\ & (\forall V0P \in ((2^{A_{\text{27b}}})^{A_{\text{27a}}})). ((\forall V1x \in A_{\text{27a}}. (\exists V2y \in \\ A_{\text{27b}}. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{\text{27b}})^{A_{\text{27a}}}). (\\ \forall V4x \in A_{\text{27a}}. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \end{aligned} \quad (78)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}. ((ap (c_{\text{2Ecombin_2EI}} \\ A_{\text{27a}}) V0x) = V0x)) \quad (79)$$

Assume the following.

$$\begin{aligned} (((ap c_{\text{2Enum_2ESUC}} c_{\text{2Earithmetic_2EZERO}}) = (ap c_{\text{2Earithmetic_2EBIT1}} \\ c_{\text{2Earithmetic_2EZERO}})) \wedge ((\forall V0n \in ty_{\text{2Enum_2Enum}}. ((ap \\ c_{\text{2Enum_2ESUC}} (ap c_{\text{2Earithmetic_2EBIT1}} V0n)) = (ap c_{\text{2Earithmetic_2EBIT2}} \\ V0n))) \wedge (\forall V1n \in ty_{\text{2Enum_2Enum}}. ((ap c_{\text{2Enum_2ESUC}} (ap c_{\text{2Earithmetic_2EBIT2}} \\ V1n)) = (ap c_{\text{2Earithmetic_2EBIT1}} (ap c_{\text{2Enum_2ESUC}} V1n))))))) \end{aligned} \quad (80)$$

Assume the following.

$((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m))))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m))))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))) \wedge ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m))))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n))))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n))))))) \wedge ((\forall V19n \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum.((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m))))))) \wedge ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False))) \wedge ((\forall V24n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n))))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.((\forall V26m \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V25n) c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m))))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge ((\forall V28n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E V28n) c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n))))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.((\forall V30m \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C V29n) c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V30m)))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C V29n) V30m))))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge ((\forall V32n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow False))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C c_2Enum_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earthmetic_2EZERO = (ap c_2Earthmetic_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
& (((ap c_2Earthmetic_2EBIT1 V0n) = c_2Earthmetic_2EZERO) \Leftrightarrow \\
& False) \wedge ((c_2Earthmetic_2EZERO = (ap c_2Earthmetic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap c_2Earthmetic_2EBIT2 V0n) = c_2Earthmetic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap c_2Earthmetic_2EBIT1 V0n) = (ap c_2Earthmetic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earthmetic_2EBIT2 V0n) = (ap c_2Earthmetic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c_2Earthmetic_2EBIT1 V0n) = (ap c_2Earthmetic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Earthmetic_2EBIT2 V0n) = (ap c_2Earthmetic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))) \\
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
 & ((p (ap (ap c_2Earthmetic_2E_3C_3D c_2Earthmetic_2EZERO) V0n)) \Leftrightarrow \\
 & True) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT1 \\
 & V0n)) c_2Earthmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D \\
 & (ap c_2Earthmetic_2EBIT2 V0n)) c_2Earthmetic_2EZERO)) \Leftrightarrow False) \wedge \\
 & (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT1 \\
 & V0n)) (ap c_2Earthmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D \\
 & V0n) V1m))) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT1 \\
 & V0n)) (ap c_2Earthmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D \\
 & V0n) V1m))) \wedge (((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT2 \\
 & V0n)) (ap c_2Earthmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earthmetic_2E_3C_3D \\
 & V1m) V0n))) \wedge ((p (ap (ap c_2Earthmetic_2E_3C_3D (ap c_2Earthmetic_2EBIT2 \\
 & V0n)) (ap c_2Earthmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earthmetic_2E_3C_3D \\
 & V0n) V1m)))))))))))
 \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \quad (85)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (86)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (87)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (88)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (89)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (90)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (91)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((\neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (93)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q) \vee (\neg(p V0p)))))))) \quad (94)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (95)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (96)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (97)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (98)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (99)$$

Theorem 1

$$\begin{aligned} \forall A_27a.\text{nonempty } A_27a \Rightarrow & (((\text{ap } (\text{c_2Ebag_2EBAG_CARD } A_27a) \\ & (\text{c_2Ebag_2EEMPTY_BAG } A_27a)) = \text{c_2Enum_2E0}) \wedge (\forall V0b \in (\\ & \text{ty_2Enum_2Enum}^{A_27a}).((\text{p } (\text{ap } (\text{c_2Ebag_2EFINITE_BAG } A_27a) \\ & V0b)) \Rightarrow (\forall V1e \in A_27a.((\text{ap } (\text{c_2Ebag_2EBAG_CARD } A_27a) (\\ & \text{ap } (\text{ap } (\text{c_2Ebag_2EBAG_INSERT } A_27a) V1e) V0b)) = (\text{ap } (\text{ap } \text{c_2Earithmetic_2E_2B} \\ & (\text{ap } (\text{c_2Ebag_2EBAG_CARD } A_27a) V0b)) (\text{ap } \text{c_2Earithmetic_2ENUMERAL} \\ & (\text{ap } \text{c_2Earithmetic_2EBIT1 } \text{c_2Earithmetic_2EZERO))))))) \end{aligned}$$