thm_2Ebag_2EBAG__DELETE__11 (TMHHmTQUSSWq9FZV153zhzzy4zudtXwL8SZ)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (5)$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 14 *We define* c_2Ebag_2EBAG__INSERT *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2E$

Definition 15 *We define* c_2Ebag_2EBAG__DELETE *to be* $\lambda A\_27a : \iota.\lambda V0b0 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2x \in A\_27a.(((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V2x)\ V0b1) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V2x)\ V1b2)) \Leftrightarrow (V0b1 = V1b2))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2b \in (ty\_2Enum\_2Enum^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0x)\ V2b) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V1y)\ V2b)) \Leftrightarrow (V0x = V1y))))) \tag{8}$$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \quad (13)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \quad (14)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b0 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1e1 \in A\_27a.(\forall V2e2 \in A\_27a.(\forall V3b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V4b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DELETE}\ A\_27a)\ V0b0)\ V1e1)\ V3b1)) \wedge (p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DELETE}\ A\_27a)\ V0b0)\ V2e2)\ V4b2))) \Rightarrow ((V1e1 = V2e2) \Leftrightarrow (V3b1 = V4b2))))))))$$