thm_2Ebag_2EBAG__DIFF__EMPTY (TM-RdR59hzQVws2Cwh8LMBtsct82g9NGnbN2)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 4 *We define* c_2Ebag_2EEMPTY__BAG *to be* $\lambda A_27a : \iota.(ap\ ($c_2Ecombin_2EK ty_2Enum_2Enu

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{4}$$

Definition 5 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 6 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 7 *We define* c_2Ebag_2EBAG__DIFF *to be* $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b2$

Definition 8 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 9 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 10 *We define* $\mathsf{c\_2Ebool\_2E\_7E}$ *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 11 *We define* $\mathsf{c\_2Ebool\_2E\_2F\_5C}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{6}$$

Definition 12 *We define* $\mathsf{c\_2Enum\_2ESUC}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 13 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$*.*

Definition 14 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 15 *We define* $\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 16 *We define* $\mathsf{c\_2Ebool\_2E\_5C\_2F}$ *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 17 *We define* $\mathsf{c\_2Earithmetic\_2E\_3C\_3D}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 18 *We define* $\mathsf{c\_2Earithmetic\_2E\_3E}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 19 *We define* $\mathsf{c\_2Earithmetic\_2E\_3E\_3D}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 20 *We define* $\mathsf{c\_2Ebag\_2EBAG\_\_INN}$ *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 21 *We define* $\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$ *to be* $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m))) \tag{7}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0}) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))))) \tag{8}$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0c)\ V0c) = \mathsf{c\_2Enum\_2E0})) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V0b1)\ V1b2)) \Leftrightarrow (\forall V2x \in A\_27a.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ V0b1\ V2x))\ (ap\ V1b2\ V2x)))))))) \tag{10}$$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap\ V0f\ V2x) = (ap\ V1g\ V2x)))))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{17}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{18}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \tag{20}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow ((\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27a)\ V0b)\ V0b) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a))) \wedge ((\forall V1b \in (ty\_2Enum\_2Enum^{A\_27b}). ((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27b)\ V1b)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27b)) = V1b)) \wedge ((\forall V2b \in (ty\_2Enum\_2Enum^{A\_27c}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27c)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c))\ V2b) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c))) \wedge (\forall V3b1 \in (ty\_2Enum\_2Enum^{A\_27d}). (\forall V4b2 \in (ty\_2Enum\_2Enum^{A\_27d}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27d)\ V3b1)\ V4b2)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27d)\ V3b1)\ V4b2) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27d))))))))$$