thm_2Ebag_2EBAG__DIFF__EMPTY__simple (TMLegeRC-thYY9QbyqPSSpVVZUXQxZo6oMzx)

October 26, 2020

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 15 *We define* c_2Ebag_2EBAG__INN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 16 *We define* c_2Ebag_2ESUB__BAG *to be* $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{5}$$

Definition 17 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 18 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 19 *We define* c_2Ebag_2EEMPTY__BAG *to be* $\lambda A_27a : \iota.(ap\ ($c_2Ecombin_2EK ty_2Enum_2En

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 20 *We define* c_2Ebag_2EBAG__DIFF *to be* $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b$

Assume the following.

$$\begin{gathered}\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.\\ nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow ((\forall V0b \in (\\ ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27a)\ V0b)\\ V0b) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a))) \wedge ((\forall V1b \in (ty\_2Enum\_2Enum^{A\_27b}).\\ ((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27b)\ V1b)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\\ A\_27b)) = V1b)) \wedge ((\forall V2b \in (ty\_2Enum\_2Enum^{A\_27c}).((ap\ (ap\\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27c)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c))\ V2b) =\\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c))) \wedge (\forall V3b1 \in (ty\_2Enum\_2Enum^{A\_27d}).\\ (\forall V4b2 \in (ty\_2Enum\_2Enum^{A\_27d}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\\ A\_27d)\ V3b1)\ V4b2)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27d)\ V3b1)\\ V4b2) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27d)))))))\end{gathered} \tag{7}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow ((\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27a)\ V0b)\ V0b) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a))) \wedge ((\forall V1b \in (ty\_2Enum\_2Enum^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27b)\ V1b)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27b)) = V1b)) \wedge (\forall V2b \in (ty\_2Enum\_2Enum^{A\_27c}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27c)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c))\ V2b) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c)))))$$