

thm_2Ebag_2EBAG__DISJOINT__BAG__IN
(TMLmw74FsDbdJBmWvRtp3ssCDcdn3FT17kr)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmic_E$

Definition 8 We define $c_Earithmic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define c_Ebool_EF to be $(ap (c_Ebool_E21 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_E21 2))$

Definition 12 We define $c_Ebool_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.V2t)))$

Definition 13 We define c_Emin_E40 to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$).

Definition 14 We define c_Ebool_E3F to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_Emin_E40 A_27a P))))$

Definition 15 We define $c_Eprim_rec_E3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$

Definition 16 We define $c_Earithmic_E3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$

Definition 17 We define $c_Ebool_E5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E21 2) (\lambda V2t \in 2.V2t)))$

Definition 18 We define $c_Earithmic_E3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V1n$

Definition 19 We define $c_Ebag_EBAG_INN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1n \in ty_2Enum_2Enum.V1n$

Definition 20 We define $c_Ebag_EBAG_IN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).V1b$

Definition 21 We define $c_Ebag_ESET_OF_BAG$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).V0b$

Definition 22 We define $c_Epred_set_EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_EF)$.

Definition 23 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \tag{7}$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \tag{8}$$

Definition 24 We define c_2Epair_E2C to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_E2C A_27a A_27b V0x V1y))$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \end{aligned} \quad (9)$$

Definition 25 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ V0s)\ V1t)$

Definition 26 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_2EGSPEC\ V0s)\ V1t)$

Definition 27 We define $c_2Ebag_2EBAG_DISJOINT$ to be $\lambda A_27a : \iota.\lambda V0b1 \in (ty_2Enum_2Enum^{A_27a})$.

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1b \in \\ (ty_2Enum_2Enum^{A_27a}).((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x) \\ (ap\ (c_2Ebag_2ESET_OF_BAG\ A_27a)\ V1b))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ebag_2EBAG_IN \\ A_27a)\ V0x)\ V1b)))))) \end{aligned} \quad (10)$$

Assume the following.

$$True \quad (11)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (12)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ p\ V0t)))))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B)))))))) \quad (18)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27a}). ((V0s = V1t) \Leftrightarrow (\forall V2x \in A.27a. ((p (ap (ap (c.2Ebool.2EIN A.27a) V2x) V0s)) \Leftrightarrow (p (ap (ap (c.2Ebool.2EIN A.27a) V2x) V1t)))))))) \quad (19)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\neg(p (ap (ap (c.2Ebool.2EIN A.27a) V0x) (c.2Epred_set.2EEMPTY A.27a)))))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27a}). (\forall V2x \in A.27a. ((p (ap (ap (c.2Ebool.2EIN A.27a) V2x) (ap (ap (c.2Epred_set.2EINTER A.27a) V0s) V1t))) \Leftrightarrow ((p (ap (ap (c.2Ebool.2EIN A.27a) V2x) V0s)) \wedge (p (ap (ap (c.2Ebool.2EIN A.27a) V2x) V1t)))))))) \quad (21)$$

Theorem 1

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0b1 \in (ty.2Enum.2Enum^{A.27a}). (\forall V1b2 \in (ty.2Enum.2Enum^{A.27a}). ((p (ap (ap (c.2Ebag.2EBAG_DISJOINT A.27a) V0b1) V1b2)) \Leftrightarrow (\forall V2e \in A.27a. ((\neg(p (ap (ap (c.2Ebag.2EBAG_IN A.27a) V2e) V0b1))) \vee (\neg(p (ap (ap (c.2Ebag.2EBAG_IN A.27a) V2e) V1b2))))))))))$$