

thm\_2Ebag\_2EBAG\_\_GEN\_\_PROD\_\_TAILREC  
 (TMKCGETVnjBn-  
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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Earithmic\_2EZERO\ c\_2Enum\_2E0))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V0t2)))$

**Definition 12** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 13** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.V0t2)))$

**Definition 14** We define  $c\_2Ebag\_2EBAG\_INSERT$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum^{A\_27a})$

**Definition 15** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

**Definition 16** We define  $c\_2Ebag\_2EEMPTY\_BAG$  to be  $\lambda A\_27a : \iota.(ap (c\_2Ecombin\_2EK ty\_2Enum\_2Enum^{A\_27a}))$

**Definition 17** We define  $c\_2Ebag\_2EFINITE\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(ap (c\_2Ecombin\_2EK ty\_2Enum\_2Enum^{A\_27a}))$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (7)$$

Let  $c\_2Ebag\_2EITBAG : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Ebag\_2EITBAG \\ A\_27a A\_27b \in (((A\_27b^{A\_27b})^{(ty\_2Enum\_2Enum^{A\_27a})})^{((A\_27b^{A\_27b})^{A\_27a})}) \end{aligned} \quad (8)$$

**Definition 18** We define  $c\_2Ebag\_2EBAG\_GEN\_PROD$  to be  $\lambda V0bag \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum^{A\_27a}})$

**Definition 19** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum. \\ (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n) = (ap (ap c\_2Earithmetic\_2E\_2A \\ V1n) V0m)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad \forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A V0m) \\
& (ap (ap c\_2Earithmetic\_2E\_2A V1n) V2p)) = (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V2p))))))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \quad \forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}). (\forall V1b \in (ty\_2Enum\_2Enum)^{A\_27a}). \\
& \quad ((\forall V2x \in A\_27a. (\forall V3y \in A\_27a. (\forall V4z \in A\_27b. \\
& ((ap (ap V0f V2x) (ap (ap V0f V3y) V4z)) = (ap (ap V0f V3y) (ap (ap V0f \\
& \quad V2x) V4z)))))) \wedge (p (ap (c\_2Ebag\_2EFINITE\_BAG A\_27a) V1b))) \Rightarrow ( \\
& \quad \forall V5x \in A\_27a. (\forall V6a \in A\_27b. ((ap (ap (ap (c\_2Ebag\_2EITBAG \\
& \quad A\_27a A\_27b) V0f) (ap (ap (c\_2Ebag\_2EBAG\_INSERT A\_27a) V5x) V1b)) \\
& \quad V6a) = (ap (ap (ap (c\_2Ebag\_2EITBAG A\_27a A\_27b) V0f) V1b) (ap (ap \\
& \quad V0f V5x) V6a)))))))))
\end{aligned} \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{13}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\
& \quad p V0t))))))
\end{aligned} \tag{14}$$

### Theorem 1

$$\begin{aligned}
& (\forall V0b \in (ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum}). ((p (ap (c\_2Ebag\_2EFINITE\_BAG \\
& \quad ty\_2Enum\_2Enum) V0b)) \Rightarrow (\forall V1x \in ty\_2Enum\_2Enum. (\forall V2a \in \\
& ty\_2Enum\_2Enum. ((ap (ap c\_2Ebag\_2EBAG\_GEN\_PROD (ap (ap (c\_2Ebag\_2EBAG\_INSERT \\
& \quad ty\_2Enum\_2Enum) V1x) V0b)) V2a) = (ap (ap c\_2Ebag\_2EBAG\_GEN\_PROD \\
& \quad V0b) (ap (ap c\_2Earithmetic\_2E\_2A V1x) V2a))))))
\end{aligned}$$