

thm_2Ebag_2EBAG__IMAGE__COMPOSE
(TMWU_{pdHNaHMb19eudFhxeWeY1Hq5z7kUxFm})

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o(x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.V0x)$

Definition 4 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A.\lambda a : \iota.(ap\ (c_2Ecombin_2EK\ ty_2Enum_2Enum))$

Definition 5 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 6 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Ecombin_2EK\ ty_2Enum_2Enum))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B V0n) V0n)$.

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$.

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A)$) of type $\iota \Rightarrow \iota$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21 2) V1t1) V2t2) V0t) V0t)$.

Definition 16 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EBAG_INSERT A_27a) V0e V1b)$.

Definition 17 We define $c_2Ebag_2EBAG_FILTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EBAG_FILTER A_27a) V0P V1b)$.

Let $c_2Ebag_2EBAG_CARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebag_2EBAG_CARD A_27a \in (ty_2Enum_2Enum^{(ty_2Enum_2Enum^{A_27a})}) \quad (7)$$

Definition 18 We define $c_2Ebag_2EFINITE_BAG$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EBAG_CARD A_27a) V0b)$.

Definition 19 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.(ap (c_2Ebool_2ELET A_27a) V0f V1x)))$.

Definition 20 We define $c_2Ebag_2EBAG_IMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27a^{A_27b}).\lambda V1b \in (ty_2Enum_2Enum^{A_27b}).(ap (c_2Ebag_2EBAG_IMAGE A_27a) V0f V1b)$.

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2) V0t)$.

Definition 22 We define $c_2Ecombin_2Eo$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in (A_27b^{A_27c}).\lambda V1g \in (A_27c^{A_27a}).(ap (c_2Ecombin_2Eo A_27a) V0f V1g)$.

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Enum_2Enum^{A_27a})}). \\ & (((p (ap V0P (c_2Ebag_2EEMPTY_BAG A_27a))) \wedge (\forall V1b \in (ty_2Enum_2Enum^{A_27a}). \\ & (((p (ap (c_2Ebag_2EFINITE_BAG A_27a) V1b)) \wedge (p (ap V0P V1b)))) \Rightarrow \\ & (\forall V2e \in A_27a.(p (ap V0P (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) \\ & V2e) V1b)))))) \Rightarrow (\forall V3b \in (ty_2Enum_2Enum^{A_27a}).((p (ap \\ & (c_2Ebag_2EFINITE_BAG A_27a) V3b)) \Rightarrow (p (ap V0P V3b)))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b \\ & \quad A_27a)\ V0f)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)) = (c_2Ebag_2EEMPTY_BAG \\ & \quad A_27b))) \end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0b \in (ty_2Enum_2Enum^{A_27a}). (\forall V1f \in (A_27b^{A_27a}). \\ & \quad (\forall V2e \in A_27a. ((p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_27a)\ V0b)) \Rightarrow \\ & \quad ((ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b\ A_27a)\ V1f)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT \\ & \quad A_27a)\ V2e)\ V0b)) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_27b)\ (ap\ V1f \\ & \quad V2e))\ (ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b\ A_27a)\ V1f)\ V0b)))))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1b \in (ty_2Enum_2Enum^{A_27a}). \\ & \quad ((p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_27a)\ V1b)) \Rightarrow (p\ (ap\ (c_2Ebag_2EFINITE_BAG \\ & \quad A_27b)\ (ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b\ A_27a)\ V0f)\ V1b)))))) \end{aligned} \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{13}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & \quad (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & \quad (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{14}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & \quad True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & \quad (p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \end{aligned} \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\
& ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in \\
& 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))) \Rightarrow \\
& (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27}))))))
\end{aligned} \tag{20}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\
& nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}). \\
& (\forall V2x \in A_{.27c}.((ap (ap (ap (c_{.2Ecombin_2Eo} A_{.27c} A_{.27b} A_{.27a}) \\
& V0f) V1g) V2x) = (ap V0f (ap V1g V2x))))))
\end{aligned} \tag{21}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\
& nonempty A_{.27c} \Rightarrow (\forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1g \in (A_{.27a}^{A_{.27c}}). \\
& (\forall V2b \in (ty_{.2Enum_2Enum}^{A_{.27c}}).((p (ap (c_{.2Ebag_2EFINITE_BAG} \\
& A_{.27c}) V2b)) \Rightarrow ((ap (ap (c_{.2Ebag_2EBAG_IMAGE} A_{.27b} A_{.27c}) (ap (\\
& ap (c_{.2Ecombin_2Eo} A_{.27c} A_{.27b} A_{.27a}) V0f) V1g)) V2b) = (ap (ap (c_{.2Ebag_2EBAG_IMAGE} \\
& A_{.27b} A_{.27a}) V0f) (ap (ap (c_{.2Ebag_2EBAG_IMAGE} A_{.27a} A_{.27c}) V1g) \\
& V2b))))))
\end{aligned}$$