

thm_2Ebag_2EBAG_IMAGE_EMPTY (TMHx1d59jfTRBAD3BXrkgKEcPzLuU4cakfk)

October 26, 2020

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 7 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmic_2E0n) V0n)$.

Definition 8 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t)))$.

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) then (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 13 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21) 2) (\lambda V2t3 \in 2.V2t3))))$.

Definition 14 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EBAG_INSERT) V0e V1b)$.

Definition 15 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$.

Definition 16 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota.(ap (c_2Ecombin_2EK) ty_2Enum_2Enum^{A_27a})$.

Definition 17 We define $c_2Ebag_2EBAG_FILTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EBAG_FILTER) V0P V1b)$.

Let $c_2Ebag_2EBAG_CARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (c_2Ebag_2EBAG_CARD A_27a \in (ty_2Enum_2Enum^{(ty_2Enum_2Enum^{A_27a})})) \quad (7)$$

Definition 18 We define $c_2Ebag_2EFINITE_BAG$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EFINITE_BAG) V0b)$.

Definition 19 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27a.(ap (c_2Ebool_2ELET) V0f V1x)))$.

Definition 20 We define $c_2Ebag_2EBAG_IMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27a^{A_27b}).\lambda V1b \in (ty_2Enum_2Enum^{A_27b}).(ap (c_2Ebag_2EBAG_IMAGE) V0f V1b)$.

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$.

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow ((c_2Ebag_2EEMPTY_BAG A_27a) = (\lambda V0x \in A_27a.c_2Enum_2E0)) \quad (8)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow ((p (ap (c_2Ebag_2EFINITE_BAG A_27a) (c_2Ebag_2EEMPTY_BAG A_27a))) \wedge (\forall V0e \in A_27a.(\forall V1b \in (ty_2Enum_2Enum^{A_27a}).((p (ap (c_2Ebag_2EFINITE_BAG A_27a) (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) V0e) V1b)))) \Leftrightarrow (p (ap (c_2Ebag_2EFINITE_BAG A_27a) V1b)))))) \quad (9)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c.2Ebag_2EBAG_CARD\ A.27a)\ (c.2Ebag_2EEMPTY_BAG\ A.27a)) = c.2Enum_2E0) \quad (10)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((ap\ (ap\ (c.2Ebag_2EBAG_FILTER\ A.27a)\ V0P)\ (c.2Ebag_2EEMPTY_BAG\ A.27a)) = (c.2Ebag_2EEMPTY_BAG\ A.27a))) \quad (11)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1b \in (ty_2Enum_2Enum^{A.27a}).((p\ (ap\ (c.2Ebag_2EFINITE_BAG\ A.27a)\ V1b)) \Rightarrow (p\ (ap\ (c.2Ebag_2EFINITE_BAG\ A.27a)\ (ap\ (ap\ (c.2Ebag_2EBAG_FILTER\ A.27a)\ V0P)\ V1b)))))) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\forall V0f \in (A.27b^{A.27a}).(\forall V1x \in A.27a.((ap\ (ap\ (c.2Ebool_2ELET\ A.27a\ A.27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \quad (14)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (17)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))) \quad (18)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\
& (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\
& V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27) \\
& V5y_27)))))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\
& A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\
& V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\
& (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2))))))
\end{aligned} \tag{20}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{A_27a}). ((ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b \\
& A_27a)\ V0f)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)) = (c_2Ebag_2EEMPTY_BAG \\
& A_27b)))
\end{aligned}$$