

thm_2Ebag_2EBAG_IMAGE_EQ_EMPTY (TMNpPCcTNDcAj1qT4AFys97Cfj5oicd83Xb)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.V0x)$

Definition 4 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A.\lambda a : \iota.(ap\ (c_2Ecombin_2EK\ ty_2Enum_2Enum))$

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 6 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))))$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Ecombin_2EK\ ty_2Enum_2Enum\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 10 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 11 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21)) (\lambda V0t \in 2.V0t)$.

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21)) (\lambda V2t \in 2.V2t)))$

Definition 14 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.V2t2)))$

Definition 16 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a})$

Definition 17 We define $c_2Ebag_2EBAG_FILTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{A_27a}).\lambda V1b \in (ty_2Enum_2Enum^{A_27a})$

Let $c_2Ebag_2EBAG_CARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebag_2EBAG_CARD A_27a \in (ty_2Enum_2Enum^{(ty_2Enum_2Enum^{A_27a})}) \quad (7)$$

Definition 18 We define $c_2Ebag_2EFINITE_BAG$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EBAG_INSERT A_27a) V0b)$

Definition 19 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b.V1x))$

Definition 20 We define $c_2Ebag_2EBAG_IMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27a^{A_27b}).\lambda V1b \in (ty_2Enum_2Enum^{A_27b})$

Definition 21 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1b \in (ty_2Enum_2Enum^{A_27a}).(\neg((ap (ap (c_2Ebag_2EBAG_INSERT A_27a) V0x) V1b) = (c_2Ebag_2EEMPTY_BAG A_27a)))))) \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Enum_2Enum^{A_27a})}). \\ & (((p (ap V0P (c_2Ebag_2EEMPTY_BAG A_27a))) \wedge (\forall V1b \in (ty_2Enum_2Enum^{A_27a}). \\ & (((p (ap (c_2Ebag_2EFINITE_BAG A_27a) V1b)) \wedge (p (ap V0P V1b)))) \Rightarrow \\ & (\forall V2e \in A_27a.(p (ap V0P (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) \\ & V2e) V1b)))))) \Rightarrow (\forall V3b \in (ty_2Enum_2Enum^{A_27a}).((p (ap \\ & (c_2Ebag_2EFINITE_BAG A_27a) V3b)) \Rightarrow (p (ap V0P V3b)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).((ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b \\ & A_27a)\ V0f)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)) = (c_2Ebag_2EEMPTY_BAG \\ & \quad A_27b))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0b \in (ty_2Enum_2Enum^{A_27a}).(\forall V1f \in (A_27b^{A_27a}). \\ & \quad (\forall V2e \in A_27a.((p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_27a)\ V0b)) \Rightarrow \\ & ((ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b\ A_27a)\ V1f)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT \\ & A_27a)\ V2e)\ V0b)) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_27b)\ (ap\ V1f \\ & V2e))\ (ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b\ A_27a)\ V1f)\ V0b)))))) \end{aligned} \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t) \Leftrightarrow (p\ V0t))) \tag{15}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \tag{18}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t))))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2) \Rightarrow (p\ V2t3)) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3))))) \quad (22)$$

Assume the following.

$$2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))) \quad (23)$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). (\forall V1f \in (A_27b^{A_27a}). ((p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_27a)\ V0b)) \Rightarrow (((ap\ (ap\ (c_2Ebag_2EBAG_IMAGE\ A_27b\ A_27a)\ V1f)\ V0b) = (c_2Ebag_2EEMPTY_BAG\ A_27b)) \Leftrightarrow (V0b = (c_2Ebag_2EEMPTY_BAG\ A_27a))))))$$