thm_2Ebag_2EBAG__IMAGE__FINITE__RESTRICTED__I (TMM8r1ZzYUopRj3EgcKhaJHz3m73BG6CPZ6)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Definition 10 *We define* c_2Ebag_2EBAG__FILTER *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1b \in (ty\_2Enum\_$

Definition 11 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 12 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 13 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 14 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Ebag\_2EBAG\_\_CARD : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebag\_2EBAG\_\_CARD\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{A\_27a})} \tag{7}$$

Definition 15 *We define* c_2Ebag_2EBAG__INSERT *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2E$

Definition 16 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 17 *We define* c_2Ebag_2EEMPTY__BAG *to be* $\lambda A\_27a : \iota.(ap$ (c_2Ecombin_2EK ty_2Enum_2En

Definition 18 *We define* c_2Ebag_2EFINITE__BAG *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(ap$

Definition 19 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Definition 20 *We define* c_2Ebag_2EBAG__IMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27a^{A\_27b}).\lambda V1b$

Definition 21 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 22 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 23 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 24 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 25 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 26 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 27 *We define* c_2Ebag_2EBAG__INN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 28 *We define* $\mathsf{c\_2Ebag\_2EBAG\_\_IN}$ *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum$

Definition 29 *We define* $\mathsf{c\_2Ebag\_2EBAG\_\_EVERY}$ *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).\lambda V1b \in (ty\_2Enum\_2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2b \in (ty\_2Enum\_2Enum^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0x)\ V2b) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V1y)\ V2b)) \Leftrightarrow (V0x = V1y))))) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Enum\_2Enum^{A\_27a})}).(((p\ (ap\ V0P\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a))) \wedge (\forall V1b \in (ty\_2Enum\_2Enum^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V1b)) \wedge (p\ (ap\ V0P\ V1b))) \Rightarrow (\forall V2e \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V2e)\ V1b))))))) \Rightarrow (\forall V3b \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V3b)) \Rightarrow (p\ (ap\ V0P\ V3b)))))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a))) \wedge (\forall V0e \in A\_27a.(\forall V1b \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0e)\ V1b))) \Leftrightarrow (p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V1b)))))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1f \in (A\_27b^{A\_27a}).(\forall V2e \in A\_27a.((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V0b)) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_IMAGE}\ A\_27b\ A\_27a)\ V1f)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V2e)\ V0b)) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27b)\ (ap\ V1f\ V2e))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_IMAGE}\ A\_27b\ A\_27a)\ V1f)\ V0b))))))) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1f \in (A\_27b^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V0b)) \Rightarrow (((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_IMAGE}\ A\_27b\ A\_27a)\ V1f)\ V0b) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27b)) \Leftrightarrow (V0b = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a)))))) \quad (12)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ((\forall V0P \in (2^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_EVERY}\ A\_27a)\ V0P)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a)))) \wedge (\forall V1P \in (2^{A\_27b}).(\forall V2e \in A\_27b.(\forall V3b \in (ty\_2Enum\_2Enum^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_EVERY}\ A\_27b)\ V1P)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27b)\ V2e)\ V3b))) \Leftrightarrow ((p\ (ap\ V1P\ V2e)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_EVERY}\ A\_27b)\ V1P)\ V3b)))))))) \tag{13}$$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{19}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{20}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{21}$$

Theorem 1

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow (\forall V0f \in (A\_27a^{A\_27a}).(\forall V1b \in (ty\_2Enum\_2Enum^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V1b)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_EVERY}\ A\_27a)\ (\lambda V2e \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ A\_27a)\ (ap\ V0f\ V2e))\ V2e)))\ V1b))) \Rightarrow ((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_IMAGE}\ A\_27a\ A\_27a)\ V0f)\ V1b) = V1b))))$$