

thm_2Ebag_2EBAG__INN__BAG__INSERT__STRONG
 (TM-
 NAq339nWt8CHXQp7otsrHBACN7TytzSKA)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap V0P (ap (c_2Emin_2E_40$ A $V0P) (c_2Emin_2E_3D A $V0P)))$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 4 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 5 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 6 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E\ 3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 7 We define $c_{\text{2Ebool_2E_21}}$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_{\text{2Emin_2E_3D}}\ (2^{A-27a})\ V)\ P)\ 0)$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ 0\ m)$

Let c_2 be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 9 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT1\ n)\ V)$

Definition 10 We define $c_2Earthmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 11 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 12 We define $c_2 \in \text{min_3D_3D_3E}$ to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o} (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 13 We define $c_{\text{c_Ebool_2E_2F_5C}}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_{\text{c_Ebool_2E_21}} 2) (\lambda V2t \in$

Definition 14 We define c_2Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 15 We define $c_2EBag_2EBAG_INSERT$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1b \in (ty_2Enum_2E$

Definition 16 We define $c_{\text{Ebool_7E}}$ to be $(\lambda V0t \in 2.(ap (ap c_{\text{Emin_3D_3D_3E}} V0t) c_{\text{Ebool_2E}}))$

Definition 17 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 18 We define $c_2Earthmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 19 We define $c_2\text{-Ebool-2E-5C-2F}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2\text{-Ebool-2E-21}\ 2)\ (\lambda V2t \in$

Definition 20 We define \in 2Earithmetic 2E 3E 3D to be $\lambda V0m \in tu$ 2Enum 2Enum 2Enum. $\lambda V1n \in tu$ 2Enum 2Enum.

Definition 21 We define $c : \text{2Ebag} \rightarrow \text{2EBAG_INN}$ to be $\lambda A. \lambda g : \lambda V0e \in A. \lambda g. \lambda V1n \in \text{tu}. \text{2Enum} \rightarrow \text{2Enum}$

Let c be a real number. Let $f(x) = x^2 + c$. Let $EVEN$ be given. Assume the following:

$$e \cdot 2E\text{arithmetic_}2EVEN \in (2^{ty \cdot 2E\text{num_}2E\text{num}}) \quad (7)$$

metric_2EODD: ι be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{tg_2Ename_2Ename})$$

We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 23 We define $c_2Earithmetic_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 24 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2EiZ V0n))$

Definition 25 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2EiZ V0m) (ap (ap c_2Earithmetic_2EiZ V1n)))$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n))))) \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V1n) V0m)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D \\ & c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D \\ & V0m) c_2Enum_2E0) = V0m))) \end{aligned} \quad (16)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap c_2Enum_2ESUC V0m)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m)) \quad (17)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\ & ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ & (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\ & ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ & (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ & V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ & (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ & V0m) V1n))))))) \end{aligned} \quad (18)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\ \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (19)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n))) \quad (20)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1e \in A_27a. (\forall V2n \in ty_2Enum_2Enum. (\forall V3m \in \\ & ty_2Enum_2Enum. (((p (ap (ap (c_2Ebag_2EBAG_INN A_27a) V1e) \\ & V2n) V0b)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V3m) V2n))) \Rightarrow (p (ap (ap \\ & (c_2Ebag_2EBAG_INN A_27a) V1e) V3m) V0b))))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\forall V0n \in ty_2Enum_2Enum. (\\ & \forall V1b \in (ty_2Enum_2Enum^{A_{27a}}). (\forall V2e1 \in A_{27a}. (\forall V3e2 \in \\ & A_{27a}. ((p (ap (ap (ap (c_2EBAG_INN A_{27a}) V2e1) V0n) (ap \\ & (ap (c_2EBAG_INSERT A_{27a}) V3e2) V1b))) \Leftrightarrow (((p (ap (ap (ap \\ & (c_2EBAG_INN A_{27a}) V2e1) (ap (ap c_2Earithmetic_2E_2D \\ & V0n) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) V1b)) \wedge (V2e1 = V3e2)) \vee (p (ap (ap (ap \\ & (c_2EBAG_INN A_{27a}) V2e1) V0n) V1b))))))) \\ \end{aligned} \quad (22)$$

Assume the following.

$$True \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}. nonempty A_{27a} \Rightarrow & (\forall V0t \in 2. ((\forall V1x \in \\ & A_{27a}. (p V0t) \Leftrightarrow (p V0t))) \quad (27) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \quad (29)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\ (p V0t)) \Leftrightarrow (p V0t)))))) \quad (31) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (32)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \wedge (((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (33)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (34)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (35)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (37)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \wedge (\forall V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in A_27a.((p V0P) \wedge (p (ap V1Q V3x))))))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B)) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B)))))))))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Leftrightarrow (p\ V1t2)) \Leftrightarrow (((p\ V0t1) \Rightarrow (p\ V1t2)) \wedge ((p\ V1t2) \Rightarrow (p\ V0t1)))))) \quad (43)$$

Assume the following.

$(\forall V0n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0n) = V0n)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V3m) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge (\forall V4n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge (\forall V5n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge (\forall V6n \in ty_2Enum_2Enum. (\forall V7m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A V6n) V7m)))))) \wedge (\forall V8n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge (\forall V9n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge (\forall V10n \in ty_2Enum_2Enum. (\forall V11m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V10n) V11m)))))) \wedge (\forall V12n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V12n)))) = c_2Enum_2E0)) \wedge (\forall V13n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 V13n)))) = c_2Enum_2E0)) \wedge (\forall V14n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP V14n) c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge (\forall V15n \in ty_2Enum_2Enum. (\forall V16m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2EEEXP (ap c_2Earithmetic_2ENUMERAL V15n)) (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2EEEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \wedge (\forall V17n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V18n \in ty_2Enum_2Enum. ((ap c_2Eprim_rec_2EPRE (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge (\forall V19n \in ty_2Enum_2Enum. (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge (\forall V20n \in ty_2Enum_2Enum. ((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge (\forall V21n \in ty_2Enum_2Enum. ((\forall V22m \in ty_2Enum_2Enum. (((ap c_2Earithmetic_2ENUMERAL V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge ((\forall V23n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V23n) c_2Enum_2E0)) \Leftrightarrow False))) \wedge (\forall V24n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL V24n)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) V24n)))) \wedge (\forall V25n \in ty_2Enum_2Enum. (\forall V26m \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2ENUMERAL V25n) (ap c_2Earithmetic_2ENUMERAL V26m)))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V27n)) \Leftrightarrow False))) \wedge (\forall V28n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V28n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Earithmetic_2ENUMERAL V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V29n)) \Leftrightarrow True))) \wedge (\forall V30m \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3E c_2Enum_2E0) V30m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3E c_2Earithmetic_2ENUMERAL V30m)))) \wedge ((\forall V31n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V31n)) \Leftrightarrow True))) \wedge (\forall V32n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3D c_2Enum_2E0) V32n)) \Leftrightarrow True)))$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{46}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \\
\end{aligned} \tag{49}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p \\
& V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \\
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & (\neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (54)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (55)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (56)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (57)$$

Theorem 1

$$\begin{aligned} & \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0b \in (\text{ty_2Enum_2Enum}^{A_27a}). \\ & (\forall V1n \in \text{ty_2Enum_2Enum}. (\forall V2e1 \in A_27a. (\forall V3e2 \in \\ & A_27a. ((p (\text{ap} (\text{ap} (\text{ap} (\text{c_2EBAG_2EBAG_INN } A_27a) V2e1) V1n) (\text{ap} \\ & (\text{ap} (\text{c_2EBAG_2EBAG_INSERT } A_27a) V3e2) V0b))) \Leftrightarrow (((p (\text{ap} (\text{ap} (\text{ap} \\ & (\text{c_2EBAG_2EBAG_INN } A_27a) V2e1) (\text{ap} (\text{ap} c_2Earithmetic_2E_2D \\ & V1n) (\text{ap} c_2Earithmetic_2ENUMERAL (\text{ap} c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) V0b)) \wedge (V2e1 = V3e2)) \vee ((p (\text{ap} (\text{ap} (\text{ap} \\ & (\text{c_2EBAG_2EBAG_INN } A_27a) V2e1) V1n) V0b)) \wedge (\neg(V2e1 = V3e2)))))))))) \end{aligned}$$