thm_2Ebag_2EBAG__INSERT__UNION (TMML-jaPSNbqTfN6vEvWM2VcwEhCDgNk9LS7)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebag_2EBAG__UNION *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1c$

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{3}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{4}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 8 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 9 *We define* c_2Ebag_2EEMPTY__BAG *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Ecombin\_2EK}\ ty\_2Enum\_2Enu$

Definition 10 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{5}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{6}$$

Definition 11 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 12 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 13 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 14 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 15 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 16 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 17 *We define* c_2Ebag_2EBAG__INSERT *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2E$

Definition 18 *We define* c_2Ebag_2EEL__BAG *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.(ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_IN}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0e \in A\_27a.(\forall V1b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0e)\ V1b1))\ V2b2) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0e)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1b1)\ V2b2))) \wedge ((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0e)\ V2b2)) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0e)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1b1)\ V2b2)))))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.(\forall V2b \in (ty\_2Enum\_2Enum^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V0x)\ V2b) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V1y)\ V2b)) \Leftrightarrow (V0x = V1y))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. nonempty\ A\_27c \Rightarrow ((\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V0b)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27a)) = V0b)) \wedge ((\forall V1b \in (ty\_2Enum\_2Enum^{A\_27b}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27b)\ (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27b))\ V1b) = V1b)) \wedge (\forall V2b1 \in (ty\_2Enum\_2Enum^{A\_27c}).(\forall V3b2 \in (ty\_2Enum\_2Enum^{A\_27c}).(((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27c)\ V2b1)\ V3b2) = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c)) \Leftrightarrow ((V2b1 = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c)) \wedge (V3b2 = (\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG}\ A\_27c))))))) \tag{9}$$

Assume the following.

$$True \tag{10}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{11}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1e \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_INSERT}\ A\_27a)\ V1e)\ V0b) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ (ap\ (\mathsf{c\_2Ebag\_2EEL\_\_BAG}\ A\_27a)\ V1e))\ V0b))))$$