

thm_2Ebag_2EBAG_IN_DIVIDES

(TMSDZPJ2i29Uqi2VH9d8KU7TtG8sv7nfps7)

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Let $c_2Enum_2ZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2ABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2ABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be ($ap\ c_2Enum_2ABS_num\ c_2Enum_2ZERO_REP$).

Definition 3 We define $c_2Earithmetic_2ZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2SUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2SUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 4 We define c_2Ebool_2ET to be ($ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$).

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$.

Definition 6 We define c_2Enum_2SUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2ABS_num\ m)$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EBIT1 n) V0)$

Definition 8 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t\in 2.V0t))$.

Definition 10 We define $c_{\text{2Emin_2E_3D_3D_3E}}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_{\text{2Emin_2E_40}}$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \text{ } x)) \text{ then } (\lambda x. x \in A \wedge \text{of type } \iota \Rightarrow \iota)$.

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_\mathit{27a} : \iota.(\lambda V0P \in (2^A_\mathit{27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_\mathit{40}$

Definition 15 We define $c_2Eprim_rec_2E_{\lambda}C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 16 We define c_2 Earithmetic_2E_3E to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 18 We define c_2 Earthmetic_2E_3E_3D to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum.$

Definition 19 We define $c_2EBag_2EBAG_INN$ to be $\lambda A._27a : \iota.\lambda V0e \in A._27a.\lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2EBag_2EBAG_IN$ to be $\lambda A_{27a} : \iota. \lambda V0e \in A_{27a}. \lambda V1b \in (ty_2Enum_2Enum;$

Definition 21 We define c_2 to be $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x))$

Definition 22 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A.\lambda 27a:\iota.(ap\ (c_2Ecombin_2EK\ ty_2Enum_2En$

Definition 23 We define c_Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 24 We define $c_2EBag_EBAG_INSERT$ to be $\lambda A.\lambda 27a : \iota.\lambda V0e \in A.27a.\lambda V1b \in (ty_2Enum_2E$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Let $c_2Ebag_2EITBAG : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow c_2Ebag_2EITBAG \\ & \quad A_27a \ A_27b \in (((A_27b^A_27b)^{(ty_2Enum_2Enum^{A_27a})})^{((A_27b^A_27b)^{A_27a})}) \end{aligned} \tag{8}$$

Definition 25 We define $c_2EBag_2EBAG_GEN_PROD$ to be $\lambda V0bag \in (ty_2Enum_2Enum^{ty_2Enum} 2Enum_2Enum)$

Definition 26 We define $c \in \text{Ebag_2EFINITE_BAG}$ to be $\lambda A.27a : \iota.\lambda V0b \in (\text{ty_2Enum_2Enum}^A)^{27a}.$ (ap

Definition 27 We define $c_2Edivides_2Edivides$ to be $\lambda V0a \in ty_2Enum_2Enum. \lambda V1b \in ty_2Enum_2Enum.$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2A V0m) V1n) = (ap (ap c_2Earithmetic_2E_2A \\ & V1n) V0m)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1e1 \in A_27a. (\forall V2e2 \in A_27a. ((p (ap (ap (c_2Ebag_2EBAG_IN \\ & A_27a) V1e1) (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) V2e2) V0b))) \Leftrightarrow \\ & ((V1e1 = V2e2) \vee (p (ap (ap (c_2Ebag_2EBAG_IN A_27a) V1e1) V0b))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p (ap (ap \\ & (c_2Ebag_2EBAG_IN A_27a) V0x) (c_2Ebag_2EEMPTY_BAG A_27a))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0P \in (2^{(ty_2Enum_2Enum^{A_27a})}). \\ & (((p (ap V0P (c_2Ebag_2EEMPTY_BAG A_27a))) \wedge (\forall V1b \in (ty_2Enum_2Enum^{A_27a}). \\ & (((p (ap (c_2Ebag_2EFINITE_BAG A_27a) V1b)) \wedge (p (ap V0P V1b))) \Rightarrow \\ & (\forall V2e \in A_27a. (p (ap V0P (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) \\ & V2e) V1b))))))) \Rightarrow (\forall V3b \in (ty_2Enum_2Enum^{A_27a}). ((p (ap \\ & (c_2Ebag_2EFINITE_BAG A_27a) V3b)) \Rightarrow (p (ap V0P V3b))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0b \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). ((p (ap (c_2Ebag_2EFINITE_BAG \\ & ty_2Enum_2Enum) V0b)) \Rightarrow (\forall V1x \in ty_2Enum_2Enum. (\forall V2a \in \\ & ty_2Enum_2Enum. ((ap (ap c_2Ebag_2EBAG_GEN_PROD (ap (ap (c_2Ebag_2EBAG_INSERT \\ & ty_2Enum_2Enum) V1x) V0b)) V2a) = (ap (ap c_2Earithmetic_2E_2A \\ & V1x) (ap (ap c_2Ebag_2EBAG_GEN_PROD V0b) V2a))))))) \end{aligned} \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (17)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (18)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (20)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A_27a}).((\forall V2x \in A_27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_27a.(p (ap V1P V3x)) \vee (p V0Q))))))) \quad (21)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (22)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.(((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27))))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27)))))) \quad (23)$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum.(p (ap (ap c_2Edivides_2Edivides V0a) V0a))) \quad (24)$$

Assume the following.

$$(\forall V0a \in ty_2Enum_2Enum.(\forall V1b \in ty_2Enum_2Enum.(\forall V2c \in ty_2Enum_2Enum.((p (ap (ap c_2Edivides_2Edivides V0a) V1b)) \Rightarrow (p (ap (ap c_2Edivides_2Edivides V0a) (ap (ap c_2Earithmetic_2E_2A V1b) V2c))))))) \quad (25)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (26)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (27)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V1q)) \vee ((\neg(p V2r)) \vee ((\neg(p V1q) \vee ((\neg(p V0p))))))))))) \wedge (((p V0p) \vee ((\neg(p V1q)) \vee ((\neg(p V2r)) \vee ((\neg(p V0p))))))))))) \\ & \quad (((p V1q) \vee ((\neg(p V1q)) \vee ((\neg(p V2r)) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee ((\neg(p V2r))))))) \wedge (((p V1q) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee ((\neg(p V2r)) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \wedge (((p V1q) \vee ((p V2r) \vee ((\neg(p V0p))))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((\neg(p V1q)) \vee ((p V0p)))))))))) \quad (34)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow ((p V0p)))))) \quad (35)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (36)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (37)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (38)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (39)$$

Theorem 1

$$\begin{aligned} & (\forall V0b \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}). (\forall V1x \in \\ & ty_2Enum_2Enum. (\forall V2a \in ty_2Enum_2Enum. (((p (ap (c_2Ebag_2EFINITE_BAG \\ & ty_2Enum_2Enum) V0b)) \wedge (p (ap (ap (c_2Ebag_2EBAG_IN ty_2Enum_2Enum) \\ & V1x) V0b))) \Rightarrow (p (ap (ap c_2Edivides_2Edivides V1x) (ap (ap c_2Ebag_2EBAG_GEN_PROD \\ & V0b) V2a))))))) \end{aligned}$$