

thm_2Ebag_2EBAG__NOT__LESS__EMPTY
 (TMRLAUZVZJM-
 cUEUQyhe3HGcBzvUQRt6V2NV)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Earithmic_2EZERO\ c_2Enum_2E0\ c_2Enum_2EREP_num\ c_2Enum_2ESUC_REP))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})ty_2Enum_2Enum) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 9 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 11 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2))$

Definition 12 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 13 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then $(the (\lambda x.x \in A \Rightarrow P x))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$

Definition 16 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$

Definition 17 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 18 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.V0m$

Definition 19 We define $c_2Ebag_2EBAG_INN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1n \in ty_2Enum_2Enum.V0e$

Definition 20 We define $c_2Ebag_2EBAG_IN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).V0e$

Definition 21 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 22 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota.(ap (c_2Ecombin_2EK ty_2Enum_2Enum^{A_27a} V0e))$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.V2t2)))$

Definition 24 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^{A_27a}).V0e$

Definition 25 We define $c_2Ebag_2EBAG_UNION$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).\lambda V1c \in (ty_2Enum_2Enum^{A_27a}).V0b$

Definition 26 We define $c_2Ebag_2EFINITE_BAG$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).(ap (c_2Ebag_2EBAG_UNION V0b))$

Definition 27 We define c_2Ebag_2Emlt1 to be $\lambda A_27a : \iota.\lambda V0r \in ((2^{A_27a})^{A_27a}).\lambda V1b1 \in (ty_2Enum_2Enum^{A_27a}).V0r$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1b \in \\ & (ty_2Enum_2Enum^{A_27a}). (\neg((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_27a) \\ & V0x)\ V1b) = (c_2Ebag_2EEMPTY_BAG\ A_27a)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ & nonempty\ A_27c \Rightarrow ((\forall V0b \in (ty_2Enum_2Enum^{A_27a}). ((ap\ (ap \\ & (c_2Ebag_2EBAG_UNION\ A_27a)\ V0b)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)) = \\ & V0b)) \wedge ((\forall V1b \in (ty_2Enum_2Enum^{A_27b}). ((ap\ (ap\ (c_2Ebag_2EBAG_UNION \\ & A_27b)\ (c_2Ebag_2EEMPTY_BAG\ A_27b))\ V1b) = V1b)) \wedge (\forall V2b1 \in \\ & (ty_2Enum_2Enum^{A_27c}). (\forall V3b2 \in (ty_2Enum_2Enum^{A_27c}). \\ & (((ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A_27c)\ V2b1)\ V3b2) = (c_2Ebag_2EEMPTY_BAG \\ & A_27c)) \Leftrightarrow ((V2b1 = (c_2Ebag_2EEMPTY_BAG\ A_27c)) \wedge (V3b2 = (c_2Ebag_2EEMPTY_BAG \\ & A_27c)))))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((p\ (ap\ (c_2Ebag_2EFINITE_BAG \\ & A_27a)\ (c_2Ebag_2EEMPTY_BAG\ A_27a))) \wedge (\forall V0e \in A_27a. (\\ & \forall V1b \in (ty_2Enum_2Enum^{A_27a}). ((p\ (ap\ (c_2Ebag_2EFINITE_BAG \\ & A_27a)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_27a)\ V0e)\ V1b))) \Leftrightarrow (p\ (ap \\ & (c_2Ebag_2EFINITE_BAG\ A_27a)\ V1b)))))) \end{aligned} \quad (9)$$

Assume the following.

$$True \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\exists V1x \in \\ & A_27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \end{aligned} \quad (14)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (15)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (17)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0r \in ((2^{A_27a})^{A_27a}). (\forall V1b \in (ty_2Enum_2Enum^{A_27a}). (\neg(p (ap (ap (ap (c_2Ebag_2Emlt1 A_27a) V0r) V1b) (c_2Ebag_2EEMPTY_BAG A_27a)))))))$$