

thm_2Ebag_2EBAG__UNION__EMPTY
 (TMMTU6n7JMtV5SzU8Dd6Cu8GeLk3fQMb5xA)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x))$

Definition 4 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota. (ap\ (c_2Ecombin_2EK\ ty_2Enum_2Enum)\ c_2Eempty_BAG)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (4)$$

Definition 5 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ (c_2Emin_2E_3D\ (2^{A_27a}))\ P) Q))$

Definition 7 We define $c_2Ebag_2EBAG_UNION$ to be $\lambda A_27a : \iota. \lambda V0b \in (ty_2Enum_2Enum^{A_27a}). (\lambda V1c \in 2.V1c. (ap\ (c_2Ecombin_2EK\ ty_2Enum_2Enum)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)))$

Definition 8 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2))\ (\lambda V0t \in 2.V0t))$

Definition 9 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p\ Q)$ of type ι .

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Definition 11 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap(ap c_2Emin_2E_3D_3D_3E V0t)c_2Ebool_2E$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap(ap c_2Earithmetic_2E_2B V0m)c_2Enum_2E0) = V0m)) \quad (5)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap(ap c_2Earithmetic_2E_2B V0m)V1n) = (ap(ap c_2Earithmetic_2E_2B V1n)V0m)))) \quad (6)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((ap(ap c_2Earithmetic_2E_2B V0m)V1n) = c_2Enum_2E0) \Leftrightarrow ((V0m = c_2Enum_2E0) \wedge (V1n = c_2Enum_2E0)))) \quad (7)$$

Assume the following.

$$True \quad (8)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (9)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V0t)))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (11)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (12)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(V0x = V0x)) \quad (13)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}. ((V0x = V0x) \Leftrightarrow \text{True})) \quad (14)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & \forall A_{\text{27b}}. \text{nonempty } A_{\text{27b}} \Rightarrow \\ & \forall V0f \in (A_{\text{27b}}^{A_{\text{27a}}}). (\forall V1g \in (A_{\text{27b}}^{A_{\text{27a}}}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_{\text{27a}}. ((ap V0f V2x) = (ap V1g V2x)))))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2. (((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow \\ (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p V0t))))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} \forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow & \forall A_{\text{27b}}. \text{nonempty } A_{\text{27b}} \Rightarrow \\ & \forall V0x \in A_{\text{27a}}. (\forall V1y \in A_{\text{27b}}. ((ap (ap (\text{c_2Ecombin_2EK} \\ A_{\text{27a}} A_{\text{27b}}) V0x) V1y) = V0x))) \end{aligned} \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \quad (18)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (19)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (22)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (24)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (29)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}. nonempty A_{.27a} \Rightarrow \forall A_{.27b}. nonempty A_{.27b} \Rightarrow \forall A_{.27c}. \\ & nonempty A_{.27c} \Rightarrow ((\forall V0b \in (ty_2Enum_2Enum^{A_{.27a}}). ((ap (ap \\ & (c_2Ebag_2EBAG_UNION A_{.27a}) V0b) (c_2Ebag_2EEMPTY_BAG A_{.27a})) = \\ & V0b)) \wedge ((\forall V1b \in (ty_2Enum_2Enum^{A_{.27b}}). ((ap (ap (c_2Ebag_2EBAG_UNION \\ & A_{.27b}) (c_2Ebag_2EEMPTY_BAG A_{.27b}) V1b) = V1b)) \wedge (\forall V2b1 \in \\ & (ty_2Enum_2Enum^{A_{.27c}}). (\forall V3b2 \in (ty_2Enum_2Enum^{A_{.27c}}). ((ap (ap (c_2Ebag_2EBAG_UNION A_{.27c}) V2b1) V3b2) = (c_2Ebag_2EEMPTY_BAG \\ & A_{.27c})) \Leftrightarrow ((V2b1 = (c_2Ebag_2EEMPTY_BAG A_{.27c})) \wedge (V3b2 = (c_2Ebag_2EEMPTY_BAG \\ & A_{.27c})))))))) \end{aligned}$$