

thm_2Ebag_2EBAG__UNION_EQ_LEFT
 (TMYSCwrH1bm7aQ5qeqDXWVbYNYiJgxX5bnp)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V0Q \in 2.V0Q)))$

Definition 4 We define $c_2Ebag_2EBAG__UNION$ to be $\lambda A_27a : \iota. \lambda V0b \in (ty_2Enum_2Enum^{A_27a}). \lambda V1c \in 2.V1c$

Definition 5 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \Rightarrow p Q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m))) \quad (3)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2B V0m) \\ & V2p) = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) \Leftrightarrow (V0m = V1n)))))) \end{aligned} \quad (4)$$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in \\ & A_27a. (p V0t)) \Leftrightarrow (p V0t))) \end{aligned} \quad (6)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow \\ & True)) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in \\ & A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\ & \forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap V0f V2x) = (ap V1g V2x))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))) \end{aligned} \quad (12)$$

Theorem 1

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1c \in (ty_2Enum_2Enum^{A_27a}). (\forall V2d \in (ty_2Enum_2Enum^{A_27a}). \\ & (((ap (ap (c_2EBag_2EBAG_UNION A_27a) V0b) V1c) = (ap (ap (c_2EBag_2EBAG_UNION \\ & A_27a) V0b) V2d)) \Rightarrow (V1c = V2d))))))) \end{aligned}$$