

Definition 8 We define $c_2Epred_set_2ESUM_IMAGE$ to be $\lambda A.27a : \iota.\lambda V0f \in (ty_2Enum_2Enum^{A-27a})$.

Definition 9 We define $c_2Ebag_2EBIG_BAG_UNION$ to be $\lambda A.27a : \iota.\lambda V0sob \in (2^{(ty_2Enum_2Enum^{A-27a})})$.

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$) of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 P))))$.

Definition 12 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21) 2) (\lambda V0t \in 2.V0t)$.

Definition 13 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2EF)$.

Definition 14 We define c_2Ebool_2EIN to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A-27a}).(ap V1f V0x)))$.

Definition 15 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2$.inj_o $(p P \Rightarrow p Q)$ of type ι .

Definition 16 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t))))$.

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.V2t))))$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A-27b})^{A-27a}}) \quad (7)$$

Definition 18 We define $c_2Epair_2E_2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2Epair_2EABS_prod A.27a A.27b) (V0x V1y))$.

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epred_set_2EGSPEC A.27a A.27b \in ((2^{A-27a})^{(ty_2Epair_2Eprod A.27a 2)^{A-27b}}) \quad (8)$$

Definition 19 We define $c_2Epred_set_2EINSERT$ to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A-27a}).(ap (c_2Epair_2EABS_prod A.27a A.27a) (V0x V1s))$.

Definition 20 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap (c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21) 2) V0t))$.

Definition 21 We define $c_2Epred_set_2EDIFF$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1t \in (2^{A-27a}).(ap (c_2Epair_2EABS_prod A.27a A.27a) (V0s V1t))$.

Definition 22 We define $c_2Epred_set_2EDELETE$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).\lambda V1x \in A.27a.(ap (ap (c_2Emin_2E_3D_3D_3E V0s) c_2Ebool_2E_7E) V1x))$.

Definition 23 We define $c_2Epred_set_2EFINITE$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A-27a}).(ap (c_2Ebool_2E_21) 2) V0s)$.

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\
& nonempty\ A_27c \Rightarrow ((\forall V0b \in (ty_2Enum_2Enum^{A_27a}).((ap\ (ap \\
& (c_2Ebag_2EBAG_UNION\ A_27a)\ V0b)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)) = \\
& V0b)) \wedge ((\forall V1b \in (ty_2Enum_2Enum^{A_27b}).((ap\ (ap\ (c_2Ebag_2EBAG_UNION \\
& A_27b)\ (c_2Ebag_2EEMPTY_BAG\ A_27b))\ V1b) = V1b)) \wedge (\forall V2b1 \in \\
& (ty_2Enum_2Enum^{A_27c}).(\forall V3b2 \in (ty_2Enum_2Enum^{A_27c}). \\
& (((ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A_27c)\ V2b1)\ V3b2) = (c_2Ebag_2EEMPTY_BAG \\
& A_27c)) \Leftrightarrow ((V2b1 = (c_2Ebag_2EEMPTY_BAG\ A_27c)) \wedge (V3b2 = (c_2Ebag_2EEMPTY_BAG \\
& A_27c)))))))))
\end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Ebag_2EBIG_BAG_UNION \\
& A_27a)\ (c_2Epred_set_2EEMPTY\ (ty_2Enum_2Enum^{A_27a}))) = (c_2Ebag_2EEMPTY_BAG \\
& A_27a))
\end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0sob \in (2^{(ty_2Enum_2Enum^{A_27a})}). \\
& (\forall V1b \in (ty_2Enum_2Enum^{A_27a}).((p\ (ap\ (c_2Epred_set_2EFINITE \\
& (ty_2Enum_2Enum^{A_27a})\ V0sob)) \Rightarrow ((ap\ (c_2Ebag_2EBIG_BAG_UNION \\
& A_27a)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ (ty_2Enum_2Enum^{A_27a}) \\
& V1b)\ V0sob)) = (ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A_27a)\ V1b)\ (ap\ (c_2Ebag_2EBIG_BAG_UNION \\
& A_27a)\ (ap\ (ap\ (c_2Epred_set_2EDELETE\ (ty_2Enum_2Enum^{A_27a}) \\
& V0sob)\ V1b)))))))))
\end{aligned} \tag{11}$$

Assume the following.

$$True \tag{12}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{13}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\
A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.(((p \ V0t) \Rightarrow False) \Rightarrow (\neg(p \ V0t)))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p \ V0t)) \Rightarrow ((p \ V0t) \Rightarrow False))) \quad (18)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (19)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\ & (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\ & (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \end{aligned} \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\ & (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \end{aligned} \quad (21)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (22)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p \ V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & (2^{A.27a}).((\exists V2x \in A.27a.((p (ap V0P V2x)) \vee (p (ap V1Q V2x)))) \Leftrightarrow \\ & ((\exists V3x \in A.27a.(p (ap V0P V3x))) \vee (\exists V4x \in A.27a.(p (\\ & \quad \quad \quad ap V1Q V4x))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).((p V0P) \vee (\exists V2x \in A.27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in \\ & \quad \quad \quad A.27a.((p V0P) \vee (p (ap V1Q V3x)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\exists V2x \in A.27a.((p (ap V0P V2x)) \wedge (p V1Q))) \Leftrightarrow ((\exists V3x \in \\ & \quad \quad \quad A.27a.(p (ap V0P V3x)) \wedge (p V1Q)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (\\ & 2^{A.27a}).((\exists V2x \in A.27a.((p V0P) \wedge (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ & \quad \quad \quad V0P) \wedge (\exists V3x \in A.27a.(p (ap V1Q V3x)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (\\ & (p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee \\ & \quad \quad \quad (p V0A)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\ & p V0A)) \vee (\neg(p V1B)))) \wedge (((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ & 2.(((p V0x) \Leftrightarrow (p V1x.27)) \wedge ((p V1x.27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y.27)))) \Rightarrow \\ & \quad \quad \quad ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x.27) \Rightarrow (p V3y.27)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EEMPTY\ A_27a)))))) \quad (36)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. (\forall V2s \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V1y)\ V2s)))) \Leftrightarrow ((V0x = V1y) \vee (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2s)))))) \quad (37)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in (2^{A_27a}). ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s))) \Leftrightarrow ((ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V1s)\ V0x) = V1s)))) \quad (38)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{(2^{A_27a})}). ((p\ (ap\ V0P\ (c_2Epred_set_2EEMPTY\ A_27a))) \wedge (\forall V1s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V1s)) \wedge (p\ (ap\ V0P\ V1s)))) \Rightarrow (\forall V2e \in A_27a. ((\neg(p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V2e)\ V1s))) \Rightarrow (p\ (ap\ V0P\ (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V2e)\ V1s)))))) \Rightarrow (\forall V3s \in (2^{A_27a}). ((p\ (ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V3s)) \Rightarrow (p\ (ap\ V0P\ V3s)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \quad (43)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (44)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(\\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \vee (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (\neg(p \ V1q))) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge \\
& ((p \ V1q) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{49}$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0sob \in (2^{(ty_2Enum_2Enum^{A-27a})}). \\
& ((p \ (ap \ (c_2Epred_set_2EFINITE \ (ty_2Enum_2Enum^{A-27a}) \ V0sob)) \Rightarrow \\
& (((ap \ (c_2Ebag_2EBIG_BAG_UNION \ A_27a) \ V0sob) = (c_2Ebag_2EEMPTY_BAG \\
& A_27a)) \Leftrightarrow (\forall V1b \in (ty_2Enum_2Enum^{A-27a}). ((p \ (ap \ (ap \ (c_2Ebool_2EIN \\
& (ty_2Enum_2Enum^{A-27a}) \ V1b) \ V0sob)) \Rightarrow (V1b = (c_2Ebag_2EEMPTY_BAG \\
& A_27a))))))
\end{aligned}$$