

thm_2Ebag_2EFINITE_BAG_THM
 (TMH9Qq1NdgEbbWt7ojhMEMtJ8YKDAgGdQw4)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (1)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$\text{nonempty } ty_2Enum_2Enum \quad (2)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (3)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 4 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_2Ebool_2E_21$ to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$

Definition 6 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 7 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT1\ n)\ V)$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2 \in \text{min_3D_3D_3E}$ to be $\lambda P \in 2.\lambda Q \in 2.\text{inj_o} (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 11 We define $c_{\text{CBool}} : \mathbb{E}_{\text{CBool}}$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_{\text{CBool}}_2)_{\text{CBool}} 21) 2)) (\lambda V2t \in$

Definition 12 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \ (ap \ P \ x)) \ \text{then} \ (\lambda x. x \in A \wedge \text{of type } \iota \rightarrow \iota)$.

Definition 13 We define $c_{\cdot 2Ebool_2ECOND}$ to be $\lambda A._27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A._27a.(\lambda V2t2 \in A._27a.($

Definition 14 We define $c_2EBag_2EBAG_INSERT$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1b \in (ty_2Enum_2E$

Definition 15 We define $c_{_2EBag_2EBAG_DELETE}$ to be $\lambda A._27a : \iota.\lambda V.0b0 \in (ty_2Enum_2Enum^A)^{A_27a}.$

Definition 16 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2EMin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E))$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 19 We define $c_2Earthmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap(c_2Ebool_2E_21 2))(\lambda V2t \in$

Definition 21 We define $c_{\text{Earth}} \in \text{EArith}_E$ to be $\lambda V0m \in \text{ty}_E[\text{Enum} \rightarrow \text{Enum}] . \lambda V1n \in \text{ty}_E[\text{Enum} \rightarrow \text{Enum}] .$

Definition 22 We define $c_2EBag_2EBAG_INN$ to be $\lambda A.\lambda 27a:\iota.\lambda V0e \in A.\lambda 27a.\lambda V1n \in ty.\lambda 2Enum_2Enum$

Definition 23 We define $c_2EBag_2EBAG_IN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_2Enum_2Enum^1)$

Definition 24 We define c_2 Ecombin- $2EK$ to be $\lambda A. \exists a : \iota. \lambda A. \exists b : \iota. (\lambda V0x \in A. \exists a. (\lambda V1y \in A. \exists b. V0x = y))$

Definition 25 We define $\in \text{2Ebag } \text{2EEMPTY_BAG}$ to be $\lambda A. \exists a : \iota. (ap (\in \text{2Ecombin } \text{2EK} \text{ tu } \text{2Enum } \text{2Em}))$

Definition 26. We define $\mathcal{C}2\text{-Ebag-2EEFINITE-BAG}$ to be $\lambda A. \exists a. \exists g. \forall b \in (tu\ 2\text{-Enum}, 2\text{-Enum}^A)^{\leq 27a}$. (an

Assume the following

$\forall A \exists 27a \text{ nonempty } A \exists 27a \rightarrow (\forall V0b \in (tu \cdot 2Enum \cdot 2Enum^A \cdot 27a))$

$$(\forall V1e1 \in A_27a. (\forall V2e2 \in A_27a. ((p (ap (ap (c_2EBAG_IN A_27a) V1e1) (ap (ap (c_2EBAG_INSERT A_27a) V2e2) V0b))) \Leftrightarrow ((V1e1 = V2e2) \vee (p (ap (ap (c_2EBAG_IN A_27a) V1e1) V0b))))))) \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0x \in A_{27a}.(\forall V1b \in \\ (ty_2Enum_2Enum^{A_{27a}}).(\neg((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a}) \\ V0x)\ V1b) = (c_2Ebag_2EMPTY_BAG\ A_{27a})))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b1 \in (ty_2Enum_2Enum^{A_{27a}}). \\ (\forall V1b2 \in (ty_2Enum_2Enum^{A_{27a}}).(\forall V2x \in A_{27a}.((\\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2x)\ V0b1) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V2x)\ V1b2)) \Leftrightarrow (V0b1 = V1b2)))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b \in (ty_2Enum_2Enum^{A_{27a}}). \\ (\forall V1e1 \in A_{27a}.(\forall V2e2 \in A_{27a}.((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V1e1)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2e2)\ V0b)) = \\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2e2)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V1e1)\ V0b))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b \in (ty_2Enum_2Enum^{A_{27a}}). \\ (\forall V1e \in A_{27a}.((p\ (ap\ (ap\ (c_2Ebag_2EBAG_IN\ A_{27a})\ V1e) \\ V0b)) \Rightarrow (\exists V2b_{27} \in (ty_2Enum_2Enum^{A_{27a}}).(p\ (ap\ (ap\ (ap\ (\\ c_2Ebag_2EBAG_DELETE\ A_{27a})\ V0b)\ V1e)\ V2b_{27}))))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0a \in A_{27a}.(\forall V1M \in \\ (ty_2Enum_2Enum^{A_{27a}}).(\forall V2b \in A_{27a}.(\forall V3N \in (ty_2Enum_2Enum^{A_{27a}}). \\ (((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V0a)\ V1M) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V2b)\ V3N)) \Leftrightarrow ((V1M = V3N) \wedge (V0a = V2b)) \vee (\exists V4k \in (ty_2Enum_2Enum^{A_{27a}}). \\ ((V1M = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V2b)\ V4k)) \wedge (V3N = \\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_{27a})\ V0a)\ V4k))))))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_{27a}) \\ (c_2Ebag_2EMPTY_BAG\ A_{27a}))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b \in (ty_2Enum_2Enum^{A_{27a}}). \\ ((p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_{27a})\ V0b)) \Rightarrow (\forall V1e \in A_{27a}. \\ (p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_{27a})\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ \\ A_{27a})\ V1e)\ V0b)))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0P \in (2^{(ty_2Enum_2Enum^{A_{27a}})})) \\
& (((p (ap V0P (c_2Ebag_2EMPTY_BAG A_{27a}))) \wedge (\forall V1b \in (ty_2Enum_2Enum^{A_{27a}}))) \\
& \quad (((p (ap (c_2Ebag_2FINITE_BAG A_{27a}) V1b)) \wedge (p (ap V0P V1b))) \Rightarrow \\
& \quad (\forall V2e \in A_{27a}. (p (ap V0P (ap (ap (c_2Ebag_2EBAG_INSERT A_{27a}) \\
& \quad V2e) V1b)))))) \Rightarrow (\forall V3b \in (ty_2Enum_2Enum^{A_{27a}}). ((p (ap \\
& \quad (c_2Ebag_2FINITE_BAG A_{27a}) V3b)) \Rightarrow (p (ap V0P V3b)))))) \\
& \tag{15}
\end{aligned}$$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \tag{19}$$

Assume the following.

$$\forall A_{27a}. nonempty A_{27a} \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_{27a}. (p V0t) \Leftrightarrow (p V0t)))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{21}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{22}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& \quad (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& \quad (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
& \tag{23}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge \\
& \quad (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \\
& \quad (p V0t)) \Leftrightarrow (p V0t)))))) \\
& \tag{24}
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (26)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (29)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p (ap V0P V2x))))))) \quad (30)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).((\forall V2x \in A_27a.((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p V0P) \vee (\forall V3x \in A_27a.(p (ap V1Q V3x))))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V0A) \vee (p V1B) \vee (p V2C)))))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (34)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))) \end{aligned} \quad (36)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (39)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (40)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)))) \wedge (((p V1q) \vee ((\neg(p V2r) \vee (p V0p)))) \wedge (((p V2r) \vee ((\neg(p V1q) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \wedge (p V2r)))) \wedge (((p V1q) \wedge ((\neg(p V0p) \wedge (p V2r)))) \wedge (((p V2r) \wedge ((\neg(p V1q) \wedge (\neg(p V0p))))))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q) \vee (p V2r)))) \wedge (((p V0p) \vee ((\neg(p V2r) \vee (p V1q)))) \wedge (((p V2r) \vee ((\neg(p V1q) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow ((p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (46)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow ((p (\text{ap} (\text{c_2Ebag_2EFINITE_BAG } \\ & A_27a) (\text{c_2Ebag_2EMPTY_BAG } A_27a))) \wedge (\forall V0e \in A_27a. \\ & \forall V1b \in (\text{ty_2Enum_2Enum}^{A_27a}).((p (\text{ap} (\text{c_2Ebag_2EFINITE_BAG } \\ & A_27a) (\text{ap} (\text{ap} (\text{c_2Ebag_2EBAG_INSERT } A_27a) V0e) V1b))) \Leftrightarrow (p (\text{ap} \\ & (\text{c_2Ebag_2EFINITE_BAG } A_27a) V1b))))))) \end{aligned}$$