

# thm\_2Ebag\_2EFINITE\_SET\_OF\_BAG

(TMbPrR36Fwc3t6UEFZKcSDP5ZRhy71oNFcB)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be ( $ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP$ ).

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be ( $ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$ ).

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$ .

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$



Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Earithmetic\_2EOOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EOOD \in (2^{ty\_2Enum\_2Enum}) \quad (8)$$

**Definition 31** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap (ap (ap (ap (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2EEEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (10)$$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 32** We define  $c\_2Eenumeral\_2EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2ESUC (ap$

**Definition 33** We define  $c\_2Eenumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 34** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 35** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 36** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 37** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF)$ .

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow & \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ & A0\ A1) \end{aligned} \quad (12)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (13)$$

**Definition 38** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (14)$$

**Definition 39** We define  $c_2.Epred\_set\_2EINSERT$  to be  $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1s \in (2^{A-27a}).(ap(c_2.Epred\_set\_2EINSERT)(V0x,V1s))$

**Definition 40** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A.\_27a : \iota.\lambda V0s \in (2^A\_{27}a).(ap\ (c\_2Ebool\_2E\_21\ (2$

Assume the following.

$$\begin{aligned}
& ((\forall Vm \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge ((ap ( \\
& ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\
& V0m) V1n))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\
& V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))
\end{aligned} \tag{15}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B V1n) V0m)))) \quad (16)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))) \quad (17)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (18)$$

Assume the following.

$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)))))))))))$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p))))))) \quad (20)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V0n)))))) \quad (21)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))))))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. (\forall V2p \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p))))))) \quad (23)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m))))))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ((\neg(V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m))))))) \quad (25)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0n) = (ap (ap c\_2Earithmetic\_2E\_2B (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)) V0n)))) \quad (26)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0b \in (ty_{.2Enum\_2Enum^{A_{.27a}}}). \\ & (\forall V1e1 \in A_{.27a}.(\forall V2e2 \in A_{.27a}.((p (ap (ap (c_{.2Ebag\_2EBAG\_IN} \\ A_{.27a}) V1e1) (ap (ap (c_{.2Ebag\_2EBAG\_INSERT} A_{.27a}) V2e2) V0b)))) \Leftrightarrow \\ & ((V1e1 = V2e2) \vee (p (ap (ap (c_{.2Ebag\_2EBAG\_IN} A_{.27a}) V1e1) V0b))))))) \\ (27) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0b1 \in (ty_{.2Enum\_2Enum^{A_{.27a}}}). \\ & (\forall V1b2 \in (ty_{.2Enum\_2Enum^{A_{.27a}}}).((V0b1 = V1b2) \Leftrightarrow (\forall V2n \in \\ & ty_{.2Enum\_2Enum}.(\forall V3e \in A_{.27a}.((p (ap (ap (c_{.2Ebag\_2EBAG\_INN} \\ A_{.27a}) V3e) V2n) V0b1)) \Leftrightarrow (p (ap (ap (c_{.2Ebag\_2EBAG\_INN} A_{.27a}) \\ V3e) V2n) V1b2))))))) \\ (28) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0b \in (ty_{.2Enum\_2Enum^{A_{.27a}}}). \\ & (\forall V1e \in A_{.27a}.((p (ap (ap (c_{.2Ebag\_2EBAG\_IN} A_{.27a}) V1e) \\ V0b)) \Rightarrow (\exists V2b_{.27} \in (ty_{.2Enum\_2Enum^{A_{.27a}}}).(p (ap (ap (ap ( \\ c_{.2Ebag\_2EBAG\_DELETE} A_{.27a}) V0b) V1e) V2b_{.27}))))))) \\ (29) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0e \in A_{.27a}.(\forall V1n \in \\ & ty_{.2Enum\_2Enum}.((p (ap (ap (ap (c_{.2Ebag\_2EBAG\_INN} A_{.27a}) V0e) \\ V1n) (c_{.2Ebag\_2EEMPTY\_BAG} A_{.27a}))) \Leftrightarrow (V1n = c_{.2Enum\_2E0})))) \\ (30) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & ((ap (c_{.2Ebag\_2ESET\_OF\_BAG} A_{.27a}) \\ (c_{.2Ebag\_2EEMPTY\_BAG} A_{.27a})) = (c_{.2Epred\_set\_2EEMPTY} A_{.27a})) \\ (31) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0e \in A_{.27a}.(\forall V1b \in \\ & (ty_{.2Enum\_2Enum^{A_{.27a}}}).((ap (c_{.2Ebag\_2ESET\_OF\_BAG} A_{.27a}) \\ (ap (ap (c_{.2Ebag\_2EBAG\_INSERT} A_{.27a}) V0e) V1b)) = (ap (ap (c_{.2Epred\_set\_2EINSERT} \\ A_{.27a}) V0e) (ap (c_{.2Ebag\_2ESET\_OF\_BAG} A_{.27a}) V1b)))))) \\ (32) \end{aligned}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0b \in (ty_{.2Enum\_2Enum^{A_{.27a}}}). \\ & (((c_{.2Epred\_set\_2EEMPTY} A_{.27a}) = (ap (c_{.2Ebag\_2ESET\_OF\_BAG} \\ A_{.27a}) V0b)) \Leftrightarrow (V0b = (c_{.2Ebag\_2EEMPTY\_BAG} A_{.27a}))) \wedge (((ap (c_{.2Ebag\_2ESET\_OF\_BAG} \\ A_{.27a}) V0b) = (c_{.2Epred\_set\_2EEMPTY} A_{.27a})) \Leftrightarrow (V0b = (c_{.2Ebag\_2EEMPTY\_BAG} \\ A_{.27a})))))) \\ (33) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0P \in (2^{(ty\_2Enum\_2Enum^{A_{27a}})}). \\
& (((p (ap V0P (c_2Ebag_2EMPTY_BAG A_{27a}))) \wedge (\forall V1b \in (ty\_2Enum\_2Enum^{A_{27a}})). \\
& (((p (ap (c_2Ebag_2EFINITE_BAG A_{27a}) V1b)) \wedge (p (ap V0P V1b))) \Rightarrow \\
& (\forall V2e \in A_{27a}. (p (ap V0P (ap (ap (c_2Ebag_2EBAG_INSERT A_{27a}) \\
& V2e) V1b)))))) \Rightarrow (\forall V3b \in (ty\_2Enum\_2Enum^{A_{27a}}). ((p (ap \\
& (c_2Ebag_2EFINITE_BAG A_{27a}) V3b)) \Rightarrow (p (ap V0P V3b)))))) \\
& (34)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) (c_2Ebag_2EMPTY_BAG A_{27a}))). \wedge (\forall V0e \in A_{27a}. ( \\
& \forall V1b \in (ty\_2Enum\_2Enum^{A_{27a}}). ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) (ap (ap (c_2Ebag_2EBAG_INSERT A_{27a}) V0e) V1b))) \Leftrightarrow (p (ap \\
& (c_2Ebag_2EFINITE_BAG A_{27a}) V1b)))))) \\
& (35)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A_{27a}}). \\
& (\forall V1b2 \in (ty\_2Enum\_2Enum^{A_{27a}}). ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) (ap (ap (c_2Ebag_2EBAG_UNION A_{27a}) V0b1) V1b2))) \Leftrightarrow ((p \\
& (ap (c_2Ebag_2EFINITE_BAG A_{27a}) V0b1)) \wedge (p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) V1b2)))))) \\
& (36)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A_{27a}}). \\
& (\forall V1e \in A_{27a}. (\forall V2s \in (2^{A_{27a}}). (((ap (ap (c_2Epred_set_INSERT \\
& A_{27a}) V1e) V2s) = (ap (c_2Ebag_2ESET_OF_BAG A_{27a}) V0b)) \Leftrightarrow (\exists V3b0 \in \\
& (ty\_2Enum\_2Enum^{A_{27a}}). (\exists V4eb \in (ty\_2Enum\_2Enum^{A_{27a}}). \\
& ((V0b = (ap (ap (c_2Ebag_2EBAG_UNION A_{27a}) V4eb) V3b0)) \wedge ((V2s = \\
& (ap (c_2Ebag_2ESET_OF_BAG A_{27a}) V3b0)) \wedge ((\forall V5e_{27} \in \\
& A_{27a}. ((p (ap (ap (c_2Ebag_2EBAG_IN A_{27a}) V5e_{27}) V4eb)) \Rightarrow (V5e_{27} = \\
& V1e))) \wedge ((\neg(p (ap (ap (c_2Ebool_2EIN A_{27a}) V1e) V2s))) \Rightarrow (p (ap \\
& (c_2Ebag_2EBAG_IN A_{27a}) V1e) V4eb))))))))))) \\
& (37)
\end{aligned}$$

Assume the following.

$$True \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (41)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (42)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (43)$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow \text{False}) \Rightarrow (\neg(p V0t)))) \quad (44)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow \text{False}))) \quad (45)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \wedge (p V0t)) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge \text{False}) \Leftrightarrow \text{False}) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (46)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \vee (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \vee \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee \text{False}) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (47)$$

Assume the following.

$$(\forall V0t \in 2.(((\text{True} \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow \text{True}) \Leftrightarrow \text{True}) \wedge (((\text{False} \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow \text{True}) \wedge (((p V0t) \Rightarrow \text{False}) \Leftrightarrow (\neg(p V0t)))))) \quad (48)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg\text{True}) \Leftrightarrow \text{False}) \wedge ((\neg\text{False}) \Leftrightarrow \text{True})))) \quad (49)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (50)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \text{True})) \quad (51)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0x \in A_{\text{27a}}. (\forall V1y \in A_{\text{27a}}. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))))) \quad (53)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}}). ((\neg(\forall V1x \in A_{\text{27a}}. (p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_{\text{27a}}. (\neg(p (ap V0P V2x))))))) \quad (54)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}}). ((\neg(\exists V1x \in A_{\text{27a}}. (p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_{\text{27a}}. (\neg(p (ap V0P V2x))))))) \quad (55)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in (2^{A_{\text{27a}}}). (\forall V1Q \in 2. (((\exists V2x \in A_{\text{27a}}. (p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A_{\text{27a}}. ((p (ap V0P V3x)) \vee (p V1Q))))))) \quad (56)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_{\text{27a}}}). (((p V0P) \vee (\exists V2x \in A_{\text{27a}}. (p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A_{\text{27a}}. ((p V0P) \vee (p (ap V1Q V3x))))))) \quad (57)$$

Assume the following.

$$\forall A_{\text{27a}}. \text{nonempty } A_{\text{27a}} \Rightarrow (\forall V0Q \in 2. (\forall V1P \in (2^{A_{\text{27a}}}. ((\forall V2x \in A_{\text{27a}}. ((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A_{\text{27a}}. ((p (ap V1P V3x)) \vee (p V0Q))))))) \quad (58)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. ((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B)))))) \quad (59)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B)) \vee (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (60)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))))) \quad (61)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge (\neg(p V1B))))))) \quad (62)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (63)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))) \quad (64)$$

Assume the following.

$$(\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2R \in 2. (((p V0P) \vee (p V1Q)) \Rightarrow (p V2R)) \Leftrightarrow (((p V0P) \Rightarrow (p V2R)) \wedge ((p V1Q) \Rightarrow (p V2R)))))) \quad (65)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftrightarrow False))) \quad (66)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (p V2t3)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (67)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \quad (68)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge (((p V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V1Q) V3x\_27) \\ & V5y\_27)))))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow \forall A\_27b.\text{nonempty } A\_27b \Rightarrow ( \\ & \forall V0P \in ((2^{A\_27b})^{A\_27a}). ((\forall V1x \in A\_27a. (\exists V2y \in \\ & A\_27b. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A\_27b^{A\_27a}). ( \\ & \forall V4x \in A\_27a. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \end{aligned} \quad (70)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ A\_27a. ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) V2t1) V3t2) = V3t2)))) \\ (71) \end{aligned}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((ap (c\_2Ecombin\_2EI \\ A\_27a) V0x) = V0x)) \quad (72)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p (ap V0P c\_2Enum\_2E0)) \wedge \\ (\forall V1n \in ty\_2Enum\_2Enum. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\ V1n))))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p (ap V0P V2n)))) \\ (73) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enum\_2EiZ (ap \\
& (ap c\_2Earithmetic\_2E\_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& ((\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
(ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
c\_2Earithmetic\_2EZERO)))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. \\
& (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Enum\_2ESUC V17n)))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
(ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
(ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& ((\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
c\_2Enum\_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

(75)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earthmetic\_2EZERO = (ap c\_2Earthmetic\_2EBIT1 V0n)) \Leftrightarrow False) \wedge \\
& (((ap c\_2Earthmetic\_2EBIT1 V0n) = c\_2Earthmetic\_2EZERO) \Leftrightarrow \\
& False) \wedge ((c\_2Earthmetic\_2EZERO = (ap c\_2Earthmetic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT2 V0n) = c\_2Earthmetic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap c\_2Earthmetic\_2EBIT1 V0n) = (ap c\_2Earthmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT2 V0n) = (ap c\_2Earthmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap c\_2Earthmetic\_2EBIT1 V0n) = (ap c\_2Earthmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c\_2Earthmetic\_2EBIT2 V0n) = (ap c\_2Earthmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))) \\
\end{aligned} \tag{76}$$

Assume the following.

$(\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow True) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m))) \wedge (((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V0n)))) \wedge ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) V1m)))))))))))))))))$

Assume the following.

$$\forall A.27a.\text{nonempty } A.27a \Rightarrow (p(\text{ap}(\text{c\_2Epred\_set\_2EFINITE } A.27a) (\text{c\_2Epred\_set\_2EEMPTY } A.27a))) \quad (78)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{(2^{A\_27a})}).(( \\
& (p(ap V0P(c\_2Epred\_set\_2EEMPTY A\_27a))) \wedge (\forall V1s \in (2^{A\_27a}). \\
& (((p(ap(c\_2Epred\_set\_2EFINITE A\_27a) V1s)) \wedge (p(ap V0P V1s))) \Rightarrow \\
& (\forall V2e \in A\_27a.((\neg(p(ap(ap(c\_2Ebool\_2EIN A\_27a) V2e) V1s))) \Rightarrow \\
& (p(ap V0P(ap(ap(c\_2Epred\_set\_2EINSERT A\_27a) V2e) V1s))))))) \Rightarrow \\
& (\forall V3s \in (2^{A\_27a}).((p(ap(c\_2Epred\_set\_2EFINITE A\_27a) \\
& V3s)) \Rightarrow (p(ap V0P V3s)))))) \\
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned} \forall A_27a. \text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ (2^{A_27a}). ((p (ap (c_2\text{Epred\_set\_2EFINITE } A_27a) (ap (ap (c_2\text{Epred\_set\_2EINSERT } \\ A_27a) V0x) V1s))) \Leftrightarrow (p (ap (c_2\text{Epred\_set\_2EFINITE } A_27a) V1s)))))) \end{aligned} \quad (80)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (81)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (82)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (83)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow \\ ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (84)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (85)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow ((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge ((p V0p) \vee ((\neg(p V2r)) \vee ((\neg(p V1q)) \vee ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (86)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ (p V1q) \wedge (p V2r))) \Leftrightarrow ((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge ((p V1q) \vee \\ (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ (p V1q) \vee (p V2r))) \Leftrightarrow ((p V0p) \vee (\neg(p V1q))) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge \\ ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\ (p V1q) \Rightarrow (p V2r))) \Leftrightarrow ((p V0p) \vee (p V1q)) \wedge ((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (89)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (90)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (91)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (92)$$

### Theorem 1

$$\begin{aligned} & \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}). \\ & ((p (ap (c\_2Epred\_set\_2EFINITE A\_27a) (ap (c\_2Ebag\_2ESET\_OF\_BAG \\ & A\_27a) V0b))) \Leftrightarrow (p (ap (c\_2Ebag\_2EFINITE\_BAG A\_27a) V0b)))) \end{aligned}$$