thm_2Ebag_2ENOT__mlt__EMPTY (TMYFh-LLAdCSJR2EZvv2gHarTy6zhECAvNiG)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \qquad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \qquad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Definition 4 *We define* c_2Ebag_2EEMPTY__BAG *to be* $\lambda A\_27a : \iota.(ap\ ($c_2Ecombin_2EK$\ ty\_2Enum\_2Enu$

Definition 5 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \qquad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

Definition 6 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 7 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27}$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$

$$(6)$$

Definition 9 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 10 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 11 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 12 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 13 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \land$
of type $\iota \Rightarrow \iota$.

Definition 16 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

Definition 17 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 18 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 19 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 20 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 21 *We define* c_2Ebag_2EBAG__INN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 22 *We define* c_2Ebag_2EBAG__IN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum^{A}$

Definition 23 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 24 *We define* c_2Ebag_2EBAG__INSERT *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2E$

Definition 25 *We define* c_2Ebag_2EBAG__UNION *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1$

Definition 26 *We define* c_2Ebag_2EFINITE__BAG *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(ap$

Definition 27 *We define* c_2Ebag_2Emlt1 *to be* $\lambda A\_27a : \iota.\lambda V0r \in ((2^{A\_27a})^{A\_27a}).\lambda V1b1 \in (ty\_2Enum\_2E$

Definition 28 *We define* c_2Ebool_2EBOUNDED *to be* $(\lambda V0v \in 2.c\_2Ebool\_2ET).$

Definition 29 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0r \in ((2^{A\_27a})^{A\_27a}).$$
$$(\forall V1b \in (ty\_2Enum\_2Enum^{A\_27a}).(\neg(p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2Emlt1}\quad (7)$$
$$A\_27a)\ V0r)\ V1b)\ (\mathsf{c\_2Ebag\_2EEMPTY\_BAG}\ A\_27a))))))$$

Assume the following.

$$True \qquad (8)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1){\Rightarrow}(p\ V1t2)){\Rightarrow}(((p$$
$$V1t2){\Rightarrow}(p\ V0t1)){\Rightarrow}((p\ V0t1){\Leftrightarrow}(p\ V1t2)))))) \qquad (9)$$

Assume the following.

$$(\forall V0t \in 2.(False{\Rightarrow}(p\ V0t))) \qquad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0t \in 2.((\exists V1x \in$$
$$A\_27a.(p\ V0t)){\Leftrightarrow}(p\ V0t))) \qquad (11)$$

Assume the following.

$$(\forall V0t \in 2.(((True{\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\wedge}True){\Leftrightarrow}$$
$$(p\ V0t)){\wedge}(((False{\wedge}(p\ V0t)){\Leftrightarrow}False){\wedge}((((p\ V0t){\wedge}False){\Leftrightarrow}False){\wedge}\quad (12)$$
$$(((p\ V0t){\wedge}(p\ V0t)){\Leftrightarrow}(p\ V0t)))))))$$

Assume the following.

$$(\forall V0t \in 2.(((True{\vee}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\vee}True){\Leftrightarrow}True){\wedge}$$
$$(((False{\vee}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\vee}False){\Leftrightarrow}(p\ V0t)){\wedge}(((p\ V0t){\vee}$$
$$(p\ V0t)){\Leftrightarrow}(p\ V0t)))))))$$
$$(13)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))){\Leftrightarrow}(p\ V0t))){\wedge}(((\neg True){\Leftrightarrow}False){\wedge}$$
$$((\neg False){\Leftrightarrow}True))) \qquad (14)$$

Assume the following.

$$(\forall V0v \in 2.((p\ (ap\ \mathsf{c\_2Ebool\_2EBOUNDED}\ V0v)){\Leftrightarrow}True)) \qquad (15)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0R \in ((2^{A\_27a})^{A\_27a}).$$
$$(\forall V1x \in A\_27a.(\forall V2z \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}$$
$$A\_27a)\ V0R)\ V1x)\ V2z)){\Leftrightarrow}((p\ (ap\ (ap\ V0R\ V1x)\ V2z)){\vee}(\exists V3y \in A\_27a.$$
$$((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V1x)\ V3y)){\wedge}(p\ (ap$$
$$(ap\ V0R\ V3y)\ V2z)))))))))$$
$$(16)$$

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Theorem 1

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).$$
$$(\forall V1b \in (ty\_2Enum\_2Enum^{A\_27a}).(\neg(p \ (ap \ (ap \ (ap \ (\mathsf{c\_2Erelation\_2ETC}$$
$$(ty\_2Enum\_2Enum^{A\_27a})) \ (ap \ (\mathsf{c\_2Ebag\_2Emlt1} \ A\_27a) \ V0R)) \ V1b)$$
$$(\mathsf{c\_2Ebag\_2EEMPTY\_\_BAG} \ A\_27a))))))$$