

thm\_2Ebag\_2ESET\_\_BAG\_I  
 (TMdukfnUya1cr2zAgoY1FrnYCA6wsBLHD1F)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be ( $ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP$ ).

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be ( $ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$ ).

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$ .

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap (c\_2Earithmetic\_2EBIT1) V0t) V0t))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.(V0x)$ .

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E_21 2) (\lambda V0t \in 2.V0t)))$ .

**Definition 10** We define  $c\_2Emin\_2E_3D_3D_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_2Ebool\_2E_7E$  to be  $(\lambda V0t \in 2.(ap (ap (c\_2Emin\_2E_3D_3D_3E) V0t) c\_2Ebool\_2EF)))$

**Definition 12** We define  $c\_2Ebool\_2E_2F_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E_21 2) (\lambda V2t \in 2.(ap (c\_2Ebool\_2E_7E) V2t))))))$

**Definition 13** We define  $c\_2Emin\_2E_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (\lambda x.x \in A \wedge \dots) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Ebool\_2E_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E_40) V0P)))$

**Definition 15** We define  $c\_2Eprim\_rec\_2E_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Eprim\_rec\_2E_3C) (V0m V1n))$

**Definition 16** We define  $c\_2Earithmetic\_2E_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Earithmetic\_2E_3E) (V0m V1n))$

**Definition 17** We define  $c\_2Ebool\_2E_5C_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E_21 2) (\lambda V2t \in 2.(ap (c\_2Ebool\_2E_3F) V2t))))))$

**Definition 18** We define  $c\_2Earithmetic\_2E_3E_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Earithmetic\_2E_3E_3D) (V0m V1n))$

**Definition 19** We define  $c\_2Ebag\_2EBAG\_INN$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum.(ap (c\_2Ebag\_2EBAG\_INN) (V0e V1n))$

**Definition 20** We define  $c\_2Ebag\_2EBAG\_IN$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum^A).(\lambda V1b \in (ty\_2Enum\_2Enum^A).ap (c\_2Ebag\_2EBAG\_IN) (V0e V1b))$

**Definition 21** We define  $c\_2Ebag\_2ESET\_OF\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^A)^{A\_27a}.$

**Definition 22** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap V1f V0x)))$

**Definition 23** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap (c\_2Ebool\_2EIN) (V1t1 V2t2))))))$

**Definition 24** We define  $c\_2Ebag\_2EBAG\_OF\_SET$  to be  $\lambda A\_27a : \iota.\lambda V0P \in (2^{A\_27a}).(\lambda V1x \in A\_27a.(ap (c\_2Ebag\_2EBAG\_OF\_SET) (V0P V1x))))$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1p \in A\_27a.((p (ap (ap (c\_2Ebag\_2EBAG\_INN A\_27a) V1p) (ap (c\_2Ebag\_2EBAG\_OF\_SET A\_27a) V0P))) \Leftrightarrow (p (ap (ap (c\_2Ebool\_2EIN A\_27a) V1p) V0P)))))$$

(7)

Assume the following.

$$True \quad (8)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (9)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow \text{True})) \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.\text{nonempty } A\_27a \Rightarrow & \forall A\_27b.\text{nonempty } A\_27b \Rightarrow \\ & (\forall V0f \in (A\_27b^{A\_27a}). (\forall V1g \in (A\_27b^{A\_27a}). ((V0f = \\ & V1g) \Leftrightarrow (\forall V2x \in A\_27a. ((ap\ V0f\ V2x) = (ap\ V1g\ V2x))))))) \end{aligned} \quad (11)$$

### Theorem 1

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0s \in (2^{A\_27a}). ((ap\ (c\_2EBAG\_2ESET\_OF\_BAG\ A\_27a)\ (ap\ (c\_2EBAG\_2ESET\_OF\_SET\ A\_27a)\ V0s)) = V0s)))$$