

thm_2Ebag_2ESET_OF_BAG_EQ_EMPTY
(TMcF-
bYgTcESYKeB6RyRWdqXUcxMSW3F2QNL)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p x)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40 A_27a P))$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ *of type* ι .

Definition 5 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a} P))$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 10 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota.(ap (c_2Ecombin_2EK\ ty_2Enum_2Enum\ c_2Enum_2E0))$

Definition 11 We define $c_Earithmetic_EZERO$ to be c_Enum_E0 .

Let $c_Enum_EREP_num : \iota$ be given. Assume the following.

$$c_Enum_EREP_num \in (\omega^{ty_Enum_Enum}) \quad (4)$$

Let $c_Enum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Enum_ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Definition 12 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num)$

Let $c_Earithmetic_E_2B : \iota$ be given. Assume the following.

$$c_Earithmetic_E_2B \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \quad (6)$$

Definition 13 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmetic_E_2B))$

Definition 14 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Definition 15 We define c_Ebool_EF to be $(ap\ (c_Ebool_E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 16 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2))\ (\lambda V2t \in 2.V2t)))$

Definition 17 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.V2t2)))$

Definition 18 We define $c_Ebag_EBAG_INSERT$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_Enum_Enum)$

Definition 19 We define $c_Ebool_E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_Emin_E_3D_3D_3E\ V0t)\ c_Ebool_E_21\ 2))$

Definition 20 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 21 We define $c_Earithmetic_E_3E$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 22 We define $c_Earithmetic_E_3E_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 23 We define $c_Ebag_EBAG_INN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1n \in ty_Enum_Enum$

Definition 24 We define $c_Ebag_EBAG_IN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1b \in (ty_Enum_Enum)$

Definition 25 We define $c_Ebag_ESET_OF_BAG$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_Enum_Enum^{A_27a}).(\lambda V1f \in (ty_Enum_Enum^{A_27a}).(ap\ V1f\ V0x))$

Definition 26 We define $c_Epred_set_EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_Ebool_EF)$.

Definition 27 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 28 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27b)\ V0x\ V1y)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \quad (9)$$

Definition 29 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epair_2EABS_prod\ A_27a\ A_27a)\ V0x\ V1s)$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & ((V0b = (c_2Ebag_2EEMPTY_BAG\ A_27a)) \vee (\exists V1b0 \in (ty_2Enum_2Enum^{A_27a}). \\ & (\exists V2e \in A_27a.(V0b = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_27a)\ V2e)\ V1b0)))))) \end{aligned} \quad (10)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1b \in (ty_2Enum_2Enum^{A_27a}).(\neg((ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A_27a)\ V0x)\ V1b) = (c_2Ebag_2EEMPTY_BAG\ A_27a)))))) \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((ap\ (c_2Ebag_2ESET_OF_BAG\ A_27a)\ (c_2Ebag_2EEMPTY_BAG\ A_27a)) = (c_2Epred_set_2EEMPTY\ A_27a)) \quad (12)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0e \in A_27a.(\forall V1b \in \\ & (ty_2Enum_2Enum^{A_27a}).((ap\ (c_2Ebag_2ESET_OF_BAG\ A_27a)\ V0e)\ V1b) = (ap\ (ap\ (c_2Epred_set_2EINSERT\ A_27a)\ V0e)\ (ap\ (c_2Ebag_2ESET_OF_BAG\ A_27a)\ V1b)))))) \end{aligned} \quad (13)$$

Assume the following.

$$True \quad (14)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (16)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (20)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1s \in (2^{A_27a}).(\neg((ap (ap (c_2Epred_set_2EINSERT A_27a) V0x) V1s) = (c_2Epred_set_2EEMPTY A_27a)))))) \quad (21)$$

Theorem 1

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}).(((c_2Epred_set_2EEMPTY A_27a) = (ap (c_2Ebag_2ESET_OF_BAG A_27a) V0b)) \Leftrightarrow (V0b = (c_2Ebag_2EEMPTY_BAG A_27a))) \wedge (((ap (c_2Ebag_2ESET_OF_BAG A_27a) V0b) = (c_2Epred_set_2EEMPTY A_27a)) \Leftrightarrow (V0b = (c_2Ebag_2EEMPTY_BAG A_27a))))))$$