thm_2Ebag_2ESUB__BAG__DIFF__simple (TMKPDXWRL1ngEsGaCWPyW7T4Zoc5CP3m239)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (3)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (4)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 13 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 14 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 15 *We define* c_2Ebag_2EBAG__INN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{5}$$

Definition 16 *We define* c_2Ebag_2EBAG__UNION *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 17 *We define* c_2Ebag_2EBAG__DIFF *to be* $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b$

Definition 18 *We define* c_2Ebag_2ESUB__BAG *to be* $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V0b1)\ V1b2)) \Rightarrow (\forall V2b3 \in (ty\_2Enum\_2Enum^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27a)\ V0b1)\ V2b3))\ V1b2)))))) \wedge (\forall V3b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V4b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V5b3 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V6b4 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V4b2)\ V3b1)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V6b4)\ V5b3)) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27a)\ V3b1)\ V4b2))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27a)\ V5b3)\ V6b4))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V3b1)\ V6b4))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V4b2)\ V5b3))))))))))) \tag{7}$$

Assume the following.

$$True \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{9}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{11}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1c \in (ty\_2Enum\_2Enum^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_DIFF}\ A\_27a)\ V0b)\ V1c))\ V0b))))$$