thm_2Ebag_2ESUB__BAG__UNION (TMd8P47ZeEMV67mhbqzci8GW63kpT9HWmmn)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebag_2EBAG__UNION *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1c$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 7 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 10 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 11 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 12 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 13 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 16 *We define* c_2Ebag_2EBAG__INN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 17 *We define* c_2Ebag_2ESUB__BAG *to be* $\lambda A\_27a : \iota.\lambda V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1b$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Earithmetic\_2EODD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \tag{7}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{8}$$

Definition 18 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 19 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 20 *We define* c_2Eprim__rec_2EPRE *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Ebool_2E

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{9}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Definition 21 *We define* c_2Enumeral_2EiZ *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 22 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Definition 23 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 24 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 25 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 26 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))) \tag{12}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{13}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{14}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{15}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{16}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0m) = V0m)\wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0m)\wedge(((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n))\ V1n))\wedge((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n)))))))))) \tag{17}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{18}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))))) \tag{19}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1n))\ V0m))))) \tag{20}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\ V0n))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}). (\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V0b1)\ V1b2) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1b2)\ V0b1)))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}). (\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V0b1)\ V1b2)) \Leftrightarrow (\forall V2x \in A\_27a.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ V0b1\ V2x))\ (ap\ V1b2\ V2x))))))) \tag{23}$$

Assume the following.

$$True \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{25}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{26}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t))))))) \tag{28}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B))))))) \tag{29}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee (p\ V1B))))) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((p\ V0t) \Rightarrow False) \Leftrightarrow ((p\ V0t) \Leftrightarrow False))) \tag{31}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{32}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \quad (33)$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \mathsf{c\_2Enum\_2E0})\ V0n) = V0n))\wedge((\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ \mathsf{c\_2Enum\_2E0}) = V1n))\wedge((\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V2n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V3m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enumeral\_2EiZ}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m))))))\wedge((\forall V4n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ \mathsf{c\_2Enum\_2E0})\ V4n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V5n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V5n)\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V6n \in ty\_2Enum\_2Enum.(\forall V7m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V6n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V7m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V6n)\ V7m)))))\wedge((\forall V8n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V8n) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V9n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V9n)\ \mathsf{c\_2Enum\_2E0}) = V9n))\wedge((\forall V10n \in ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V10n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V11m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V10n)\ V11m)))))\wedge((\forall V12n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ V12n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V13n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ V13n))) = \mathsf{c\_2Enum\_2E0}))\wedge((\forall V14n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ V14n)\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))\wedge((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EEXP\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V15n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V16m)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ (ap\ c\_2Earithmetic\_2EEXP\ V15n)\ V16m)))))\wedge(((ap\ \mathsf{c\_2Enum\_2ESUC}\ \mathsf{c\_2Enum\_2E0}) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V17n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V17n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V17n))))\wedge(((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ \mathsf{c\_2Enum\_2E0}) = \mathsf{c\_2Enum\_2E0})\wedge((\forall V18n \in ty\_2Enum\_2Enum.((ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V18n)) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2EPRE}\ V18n))))\wedge((\forall V19n \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V19n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(V19n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V20n \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V20n))\Leftrightarrow(V20n = \mathsf{c\_2Earithmetic\_2EZERO})))\wedge((\forall V21n \in ty\_2Enum\_2Enum.(\forall V22m \in ty\_2Enum\_2Enum.(((ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V21n) = (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V22m))\Leftrightarrow(V21n = V22m))))\wedge((\forall V23n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V23n)\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow False))\wedge((\forall V24n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V24n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V24n))))\wedge((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V25n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V26m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V25n)\ V26m)))))\wedge((\forall V27n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ \mathsf{c\_2Enum\_2E0})\ V27n))\Leftrightarrow False))\wedge((\forall V28n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V28n))\ \mathsf{c\_2Enum\_2E0}))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Earithmetic\_2EZERO})\ V28n))))\wedge((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V29n))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ V30m)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V30m)\ V29n)))))\wedge((\forall V31n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V31n))\Leftrightarrow True))\wedge((\forall V32n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(
((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ \mathsf{c_2Earithmetic_2EZERO})\ V0n)) \Leftrightarrow
True) \wedge (((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V0n))\ \mathsf{c_2Earithmetic_2EZERO})) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}
(ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V0n))\ \mathsf{c_2Earithmetic_2EZERO})) \Leftrightarrow False) \wedge
(((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}
V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}
V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}
V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}
V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT1}\ V1m))) \Leftrightarrow (\neg (p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}
V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}
V0n))\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c_2Earithmetic_2E_3C_3D}
V0n)\ V1m))))))))))) \tag{35}$$

Theorem 1

$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ V0b1)\ V1b2)) \Rightarrow (\forall V2b \in (ty\_2Enum\_2Enum^{A\_27a}).(p\ ($
$ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V0b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V1b2)\ V2b))))))) \land ((\forall V3b1 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V4b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ V3b1)\ V4b2)) \Rightarrow (\forall V5b \in (ty\_2Enum\_2Enum^{A\_27a}).(p\ ($
$ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V3b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V5b)\ V4b2))))))) \land ((\forall V6b1 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V7b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V8b3 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V6b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V7b2)\ V8b3))) \Rightarrow (\forall V9b \in (ty\_2Enum\_2Enum^{A\_27a}).(p$
$(ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V6b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V7b2)\ V9b))\ V8b3)))))))) \land$
$((\forall V10b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V11b2 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V12b3 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ V10b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V11b2)\ V12b3))) \Rightarrow$
$(\forall V13b \in (ty\_2Enum\_2Enum^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ V10b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V13b)\ V11b2))\ V12b3)))))))) \land ((\forall V14b1 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V15b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V16b3 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V14b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V16b3)\ V15b2))) \Rightarrow (\forall V17b \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V14b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V16b3)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V15b2)\ V17b))))))))) \land$
$((\forall V18b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V19b2 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V20b3 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ V18b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V20b3)\ V19b2))) \Rightarrow$
$(\forall V21b \in (ty\_2Enum\_2Enum^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ V18b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V20b3)\ (ap\ ($
$ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V21b)\ V19b2))))))))) \land ((\forall V22b1 \in$
$(ty\_2Enum\_2Enum^{A\_27a}).(\forall V23b2 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V24b3 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V25b4 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V22b1)\ V24b3)) \Rightarrow ((p\ (ap\ ($
$ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V23b2)\ V25b4)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V22b1)\ V23b2))\ (ap$
$(ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V24b3)\ V25b4))))))))) \land ((\forall V26b1 \in$
$(ty\_2Enum\_2Enum^{A\_27a}).(\forall V27b2 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V28b3 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V29b4 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V26b1)\ V29b4)) \Rightarrow ((p\ (ap\ ($
$ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V27b2)\ V28b3)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V26b1)\ V27b2))\ (ap$
$(ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V28b3)\ V29b4))))))))) \land ((\forall V30b1 \in$
$(ty\_2Enum\_2Enum^{A\_27a}).(\forall V31b2 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V32b3 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V33b4 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V34b5 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}$
$A\_27a)\ V30b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V32b3)\ V34b5))) \Rightarrow$
$((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V31b2)\ V33b4)) \Rightarrow (p\ (ap\ (ap$
$(\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)$
$V30b1)\ V31b2))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V32b3)\ V33b4))\ V34b5)))))))))) \land ((\forall V35b1 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V36b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V37b3 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$(\forall V38b4 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V39b5 \in (ty\_2Enum\_2Enum^{A\_27a}).$
$((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ V35b1)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V38b4)\ V39b5))) \Rightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)$
$V36b2)\ V37b3)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2ESUB\_\_BAG}\ A\_27a)\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}$
$A\_27a)\ V35b1)\ V36b2))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ (ap$
$(ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V37b3)\ V38b4))\ V39b5)))))))))) \land$
$((\forall V40b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V41b2 \in (ty\_2Enum\_2Enum^{A\_27a}).$