

thm_2Ebag_2ESUB__BAG__UNION__eliminate (TMSXGY2CdQK7CijcAuAemxp88sAhvuXzik)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (2)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_7E$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})) (P) (Q))$

Definition 4 We define $c_2Ebag_2EBAG_UNION$ to be $\lambda A.\lambda a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A-27a}).\lambda V1c \in (ty_2Enum_2Enum^{A-27a}).$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 7 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 8 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (3)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (4)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (5)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge$
of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 17 We define $c_2Ebag_2EBAG_INN$ to be $\lambda A_27a : \iota.\lambda V0e \in A_27a.\lambda V1n \in ty_2Enum_2Enum$

Definition 18 We define $c_2Ebag_2ESUB_BAG$ to be $\lambda A_27a : \iota.\lambda V0b1 \in (ty_2Enum_2Enum^{A_27a}).\lambda V1b2$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad V1n)\ V0m)))) \end{aligned} \tag{6}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ & \quad \forall V2p \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n))\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & \quad V0m)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1n)\ V2p)))))) \end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b1 \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1b2 \in (ty_2Enum_2Enum^{A_27a}).((p\ (ap\ (ap\ (c_2Ebag_2ESUB_BAG \\ & \quad A_27a)\ V0b1)\ V1b2)) \Leftrightarrow (\forall V2x \in A_27a.(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & \quad (ap\ V0b1\ V2x))\ (ap\ V1b2\ V2x))))))) \end{aligned} \tag{8}$$

Assume the following.

$$True \tag{9}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ & \quad A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \end{aligned} \tag{10}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (11)
\end{aligned}$$

Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (12)$$

Theorem 1

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0b1 \in (ty_2Enum_2Enum^{A_27a}). \\
& (\forall V1b2 \in (ty_2Enum_2Enum^{A_27a}). (\forall V2b3 \in (ty_2Enum_2Enum^{A_27a}). \\
& (((p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \\
& \ A_27a) \ V0b1) \ V1b2)) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \ A_27a) \ V0b1) \\
& \ V2b3))) \Leftrightarrow (p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ V1b2) \ V2b3))) \wedge (\\
& ((p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \\
& \ A_27a) \ V0b1) \ V1b2)) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \ A_27a) \ V2b3) \\
& \ V0b1))) \Leftrightarrow (p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ V1b2) \ V2b3))) \wedge (\\
& ((p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \\
& \ A_27a) \ V1b2) \ V0b1)) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \ A_27a) \ V0b1) \\
& \ V2b3))) \Leftrightarrow (p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ V1b2) \ V2b3))) \wedge (\\
& (p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \\
& \ A_27a) \ V1b2) \ V0b1)) \ (ap \ (ap \ (c_2Ebag_2EBAG_UNION \ A_27a) \ V2b3) \\
& \ V0b1))) \Leftrightarrow (p \ (ap \ (ap \ (c_2Ebag_2ESUB_BAG \ A_27a) \ V1b2) \ V2b3)))))))))
\end{aligned}$$