thm_2Ebag_2ETC__mlt1__FINITE__BAG (TMX-PNkhGhm1MqCAvR6BujrfExuTc7gDUNue)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t)).$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 13 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 14 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 15 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 16 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 17 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 18 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 19 *We define* c_2Ebag_2EBAG__INN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 20 *We define* c_2Ebag_2EBAG__IN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum$

Definition 21 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x$

Definition 22 *We define* c_2Ebag_2EEMPTY__BAG *to be* $\lambda A\_27a : \iota.(ap$ (c_2Ecombin_2EK ty_2Enum_2En

Definition 23 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 24 *We define* c_2Ebag_2EBAG__INSERT *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2E$

Definition 25 *We define* c_2Ebag_2EBAG__UNION *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1$

Definition 26 *We define* c_2Ebag_2EFINITE__BAG *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(ap$

Definition 27 *We define* c_2Ebag_2Emlt1 *to be* $\lambda A\_27a : \iota.\lambda V0r \in ((2^{A\_27a})^{A\_27a}).\lambda V1b1 \in (ty\_2Enum\_2E$

Definition 28 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Assume the following.

$$True \tag{7}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{8}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{9}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{10}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{11}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{12}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1P \in ((2^{A\_27a})^{A\_27a}).(((\forall V2x \in A\_27a.(\forall V3y \in A\_27a.((p\ (ap\ (ap\ V0R\ V2x)\ V3y)) \Rightarrow (p\ (ap\ (ap\ V1P\ V2x)\ V3y))))) \wedge (\forall V4x \in A\_27a.(\forall V5y \in A\_27a.(\forall V6z \in A\_27a.(((p\ (ap\ (ap\ V1P\ V4x)\ V5y)) \wedge (p\ (ap\ (ap\ V1P\ V5y)\ V6z))) \Rightarrow (p\ (ap\ (ap\ V1P\ V4x)\ V6z))))))) \Rightarrow (\forall V7u \in A\_27a.(\forall V8v \in A\_27a.((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ A\_27a)\ V0R)\ V7u)\ V8v)) \Rightarrow (p\ (ap\ (ap\ V1P\ V7u)\ V8v)))))))) \tag{14}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}).(\forall V1b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Enum\_2Enum^{A\_27a}))\ (ap\ (\mathsf{c\_2Ebag\_2Emlt1}\ A\_27a)\ V0R))\ V1b1)\ V2b2)) \Rightarrow ((p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V1b1)) \wedge (p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V2b2)))))))$$