

thm\_2Ebag\_2ETC\_\_mlt1\_\_UNION2\_\_I  
(TMLwbzT7z43kr5SLSF6AjuLZ2yP9CLkgFfX)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o(x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.\lambda a : \iota.\lambda A.\lambda b : \iota.(\lambda V0x \in A.\lambda V1y \in A.V0x)$

**Definition 4** We define  $c\_2Ebag\_2EEMPTY\_BAG$  to be  $\lambda A.\lambda a : \iota.(ap\ (c\_2Ecombin\_2EK\ ty\_2Enum\_2Enum))$

**Definition 5** We define  $c\_2Earithmic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 6** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 7** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))))$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Ecombin\_2EK\ ty\_2Enum\_2Enum\ m))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (6)$$

**Definition 9** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2E\_2B))$

**Definition 10** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 11** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 12** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 13** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V0t)))$

**Definition 14** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A.\lambda V0x \in 2.V0x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 15** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.V0t)))$

**Definition 16** We define  $c\_2Ebag\_2EBAG\_INSERT$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum)^A$

**Definition 17** We define  $c\_2Ebag\_2EEL\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.(ap (ap (c\_2Ebag\_2EBAG\_INSERT)))$

**Definition 18** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_21 2))$

**Definition 19** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a})^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40)))$

**Definition 20** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 21** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 22** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2)) (\lambda V2t \in 2.V0t)))$

**Definition 23** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 24** We define  $c\_2Ebag\_2EBAG\_INN$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Ebag\_2EBAG\_IN$  to be  $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum)^A$

**Definition 26** We define  $c\_2Ebag\_2EBAG\_UNION$  to be  $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum)^{A\_27a}.\lambda V1b \in (ty\_2Enum\_2Enum)^{A\_27a}$

**Definition 27** We define  $c\_2Ebag\_2EFINITE\_BAG$  to be  $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum)^{A\_27a}.$

**Definition 28** We define  $c\_2Ebag\_2Emlt1$  to be  $\lambda A\_27a : \iota.\lambda V0r \in ((2^{A\_27a})^{A\_27a}).\lambda V1b1 \in (ty\_2Enum\_2Enum)^{A\_27a}$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (7)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (8)$$

**Definition 29** We define  $c\_Eprim\_rec\_EPRE$  to be  $\lambda V0m \in ty\_Enum\_Enum.(ap (ap (ap (c\_Ebool\_2E$   
Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (9)$$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (10)$$

Let  $c\_Earithmetic\_E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2A \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (11)$$

**Definition 30** We define  $c\_Enumeral\_EiZ$  to be  $\lambda V0x \in ty\_Enum\_Enum.V0x$ .

**Definition 31** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap (ap c\_Earithmetic$

**Definition 32** We define  $c\_Earithmetic\_E\_3C\_3D$  to be  $\lambda V0m \in ty\_Enum\_Enum.\lambda V1n \in ty\_Enum\_Enum.$

**Definition 33** We define  $c\_ERelation\_ETC$  to be  $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$   
Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_Enum\_Enum.(\forall V1n \in ty\_Enum\_Enum.( \\ & ((ap (ap c\_Earithmetic\_E\_2B c\_Enum\_E0) V0m) = V0m) \wedge (((ap ( \\ & ap c\_Earithmetic\_E\_2B V0m) c\_Enum\_E0) = V0m) \wedge (((ap (ap c\_Earithmetic\_E\_2B \\ & (ap c\_Enum\_ESUC V0m)) V1n) = (ap c\_Enum\_ESUC (ap (ap c\_Earithmetic\_E\_2B \\ & V0m) V1n))) \wedge ((ap (ap c\_Earithmetic\_E\_2B V0m) (ap c\_Enum\_ESUC \\ & V1n)) = (ap c\_Enum\_ESUC (ap (ap c\_Earithmetic\_E\_2B V0m) V1n)))))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_Enum\_Enum.(\forall V1n \in ty\_Enum\_Enum.( \\ & (ap (ap c\_Earithmetic\_E\_2B V0m) V1n) = (ap (ap c\_Earithmetic\_E\_2B \\ & V1n) V0m)))) \end{aligned} \quad (13)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_Enum\_Enum.(\forall V1n \in ty\_Enum\_Enum.( \\ & (ap (ap c\_Earithmetic\_E\_2B V0m) V1n) = (ap (ap c\_Earithmetic\_E\_2B \\ & V1n) V0m)))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_Enum\_Enum.(\forall V1n \in ty\_Enum\_Enum.( \\ & \forall V2p \in ty\_Enum\_Enum.((ap (ap c\_Earithmetic\_E\_2B V0m) \\ & (ap (ap c\_Earithmetic\_E\_2B V1n) V2p)) = (ap (ap c\_Earithmetic\_E\_2B \\ & (ap (ap c\_Earithmetic\_E\_2B V0m) V1n)) V2p)))))) \end{aligned} \quad (15)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (16)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V0m) = c\_2Enum\_2E0) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) V0m) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = V0m) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Earithmetic\_2E\_2B (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)) V1n)) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A V0m) (ap c\_2Enum\_2ESUC V1n)) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n)))))))))) \quad (17)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \forall V2p \in ty\_2Enum\_2Enum.(((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V2p)))))) \quad (18)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (V0m = V1n) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)) \wedge (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (19)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \forall V2p \in ty\_2Enum\_2Enum.(((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)) (ap (ap c\_2Earithmetic\_2E\_2B V0m) V2p))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V2p)))))) \quad (20)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V1n)) V0m)))))) \quad (21)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ V0n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ V0n))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1b \in (ty\_2Enum\_2Enum^{A\_27a}).(\neg((ap\ (ap\ (c\_2Ebag\_2EBAG\_INSERT\ A\_27a)\ V0x)\ V1b) = (c\_2Ebag\_2EEMPTY\_BAG\ A\_27a)))))) \quad (23)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V0b1)\ V1b2) = (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V1b2)\ V0b1)))))) \quad (24)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2b3 \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V0b1)\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V1b2)\ V2b3)) = (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V0b1)\ V1b2))\ V2b3)))))) \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow ((\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V0b)\ (c\_2Ebag\_2EEMPTY\_BAG\ A\_27a)) = V0b)) \wedge ((\forall V1b \in (ty\_2Enum\_2Enum^{A\_27b}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27b)\ (c\_2Ebag\_2EEMPTY\_BAG\ A\_27b))\ V1b) = V1b)) \wedge (\forall V2b1 \in (ty\_2Enum\_2Enum^{A\_27c}).(\forall V3b2 \in (ty\_2Enum\_2Enum^{A\_27c}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27c)\ V2b1)\ V3b2) = (c\_2Ebag\_2EEMPTY\_BAG\ A\_27c)) \Leftrightarrow ((V2b1 = (c\_2Ebag\_2EEMPTY\_BAG\ A\_27c)) \wedge (V3b2 = (c\_2Ebag\_2EEMPTY\_BAG\ A\_27c)))))))))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V1e \in A\_27a.((ap\ (ap\ (c\_2Ebag\_2EBAG\_INSERT\ A\_27a)\ V1e)\ V0b) = (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ (ap\ (c\_2Ebag\_2EEL\_BAG\ A\_27a)\ V1e))\ V0b)))) \quad (27)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\neg(p\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_IN\ A_{.27a})\ V0x)\ (c\_2Ebag\_2EMPTY\_BAG\ A_{.27a})))))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0P \in (2^{(ty\_2Enum\_2Enum^{A_{.27a}})}). \\ & (((p\ (ap\ V0P\ (c\_2Ebag\_2EMPTY\_BAG\ A_{.27a}))) \wedge (\forall V1b \in (ty\_2Enum\_2Enum^{A_{.27a}}). \\ & (((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{.27a})\ V1b)) \wedge (p\ (ap\ V0P\ V1b)))) \Rightarrow \\ & (\forall V2e \in A_{.27a}.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_INSERT\ A_{.27a}) \\ & V2e)\ V1b)))))) \Rightarrow (\forall V3b \in (ty\_2Enum\_2Enum^{A_{.27a}}).((p\ (ap \\ & (c\_2Ebag\_2EFINITE\_BAG\ A_{.27a})\ V3b)) \Rightarrow (p\ (ap\ V0P\ V3b)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow ((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG \\ & A_{.27a})\ (c\_2Ebag\_2EMPTY\_BAG\ A_{.27a}))) \wedge (\forall V0e \in A_{.27a}.( \\ & \forall V1b \in (ty\_2Enum\_2Enum^{A_{.27a}}).((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG \\ & A_{.27a})\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_INSERT\ A_{.27a})\ V0e)\ V1b))) \Leftrightarrow (p\ (ap \\ & (c\_2Ebag\_2EFINITE\_BAG\ A_{.27a})\ V1b)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A_{.27a}}). \\ & (\forall V1b2 \in (ty\_2Enum\_2Enum^{A_{.27a}}).((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG \\ & A_{.27a})\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A_{.27a})\ V0b1)\ V1b2))) \Leftrightarrow ((p \\ & (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{.27a})\ V0b1)) \wedge (p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG \\ & A_{.27a})\ V1b2)))))) \end{aligned} \quad (31)$$

Assume the following.

$$True \quad (32)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (33)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \quad (35)$$

Assume the following.

$$\forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_{.27a}.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (36)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (37)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
& (((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
& (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (38)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (39)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True))) \quad (40)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\
& \forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27b^{A\_27a}).((V0f = \\
& V1g) \Leftrightarrow (\forall V2x \in A\_27a.((ap \ V0f \ V2x) = (ap \ V1g \ V2x)))))) \quad (44)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p \ V0t)))))) \quad (45)
\end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}). (\forall V1Q \in \\ & 2.(((\forall V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q))) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ & 2^{A.27a}). (((p \ V0P) \wedge (\forall V2x \in A.27a.(p \ (ap \ V1Q \ V2x)))) \Leftrightarrow (\forall V3x \in \\ & A.27a.((p \ V0P) \wedge (p \ (ap \ V1Q \ V3x)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0Q \in 2. (\forall V1P \in ( \\ & 2^{A.27a}). ((\forall V2x \in A.27a.((p \ (ap \ V1P \ V2x)) \vee (p \ V0Q))) \Leftrightarrow ((\forall V3x \in \\ & A.27a.(p \ (ap \ V1P \ V3x)) \vee (p \ V0Q)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p \ V0A) \vee ( \\ & (p \ V1B) \vee (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \vee (p \ V2C)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg( \\ & p \ V0A) \vee (\neg(p \ V1B)))))) \wedge (((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A) \wedge (\neg(p \ V1B))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p \ V0A) \vee ( \\ & (p \ V1B) \wedge (p \ V2C))) \Leftrightarrow (((p \ V0A) \vee (p \ V1B)) \wedge ((p \ V0A) \vee (p \ V2C)))))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0t \in 2. (((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \quad (52)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow \\ & ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Leftrightarrow (p \ V1t2)) \Leftrightarrow (((p \\ & V0t1) \wedge (p \ V1t2)) \vee ((\neg(p \ V0t1) \wedge (\neg(p \ V1t2))))))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x.27 \in 2. (\forall V2y \in 2. (\forall V3y.27 \in \\ & 2. (((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))))) \Rightarrow \\ & (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \end{aligned} \quad (55)$$



Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap\ (ap\ (c\_2Ecombin\_2EK \\ & \quad A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D c\_2Earithmetic\_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& (ap c\_2Earithmetic\_2EBIT2 V0n)) c\_2Earithmetic\_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m)))) \wedge (((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT1 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m)))) \wedge (((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V1m) V0n)))) \wedge (((p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2EBIT2 \\
& V0n)) (ap c\_2Earithmetic\_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0n) V1m)))))))))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& ((\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p (ap (ap V0R V1x) V2y)) \Rightarrow \\
& (p (ap (ap (ap (c\_2Erelation\_2ETC A\_27a) V0R) V1x) V2y)))))) \wedge (\forall V3x \in \\
& A\_27a. (\forall V4y \in A\_27a. (\forall V5z \in A\_27a. (((p (ap (ap (ap \\
& (c\_2Erelation\_2ETC A\_27a) V0R) V3x) V4y)) \wedge (p (ap (ap (ap (c\_2Erelation\_2ETC \\
& A\_27a) V0R) V4y) V5z))) \Rightarrow (p (ap (ap (ap (c\_2Erelation\_2ETC A\_27a) \\
& V0R) V3x) V5z))))))))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a. nonempty A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\
& (\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p (ap (ap V0R V1x) V2y)) \Rightarrow \\
& (p (ap (ap (ap (c\_2Erelation\_2ETC A\_27a) V0R) V1x) V2y))))))
\end{aligned} \tag{60}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{61}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0A \in 2.((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False)) \quad (65)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg( \\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (66)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \end{aligned} \quad (67)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\ & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow ( \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (( \\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (70)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \quad (71)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (72)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (75)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & (\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}). (\forall V2b1 \in (ty\_2Enum\_2Enum^{A\_27a}). \\ & (((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A\_27a)\ V1b2)) \wedge ((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG \\ & A\_27a)\ V2b1)) \wedge (\neg(V1b2 = (c\_2Ebag\_2EEMPTY\_BAG\ A\_27a)))))) \Rightarrow (p \\ & (ap\ (ap\ (ap\ (c\_2Erelation\_2ETC\ (ty\_2Enum\_2Enum^{A\_27a}))\ (ap\ (c\_2Ebag\_2Emlt1 \\ & A\_27a)\ V0R))\ V2b1)\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A\_27a)\ V2b1)\ V1b2)))))) \end{aligned}$$