

thm_2Ebag_2EmltLT_-SING0 (TMawf856x4S7pZms3R8kFQyijHmDRtKVBWN)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1P \in 2.V1P)) (\lambda V2P \in 2.V2P)))$

Definition 4 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p \ P \Rightarrow p \ Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2EF))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. inj_o (V0t1 = V1t2))))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (2)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (3)$$

Let $c_2Enum_2EAABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EAABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (4)$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap c_2Enum_2EAABS_num m)$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p (ap P x)) \text{ then } (\text{the } (\lambda x. x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Ebag_2EBAG_INN$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (5)$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (6)$$

Definition 18 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 20 We define $c_2Ebag_2EBAG_IN$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1b \in (ty_2Enum_2Enum^{ty_2Enum_2Enum})$

Definition 21 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2.(\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 22 We define $c_2Ebag_2EBAG_INSERT$ to be $\lambda A_27a : \iota. \lambda V0e \in A_27a. \lambda V1b \in (ty_2Enum_2Enum^{ty_2Enum_2Enum})$

Definition 23 We define $c_2Ebag_2ESET_OF_BAG$ to be $\lambda A_27a : \iota. \lambda V0b \in (ty_2Enum_2Enum^{A_27a}).(\lambda V1b \in (ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})$

Definition 24 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. (\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 25 We define $c_2Ebag_2Edominates$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27b})^{A_27a}). \lambda V1s1 \in (ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (7)$$

Definition 26 We define $c_2Ebag_2EBAG_DIFF$ to be $\lambda A_27a : \iota. \lambda V0b1 \in (ty_2Enum_2Enum^{A_27a}). \lambda V1b2 \in (ty_2Enum_2Enum^{A_27a})$

Definition 27 We define $c_2Ebag_2EBAG_UNION$ to be $\lambda A_27a : \iota. \lambda V0b \in (ty_2Enum_2Enum^{A_27a}). \lambda V1b \in (ty_2Enum_2Enum^{A_27a})$

Definition 28 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota. (\lambda V0x \in A_27a. c_2Ebool_2EF)$.

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_0.nonempty A_0 \Rightarrow & \forall A_1.nonempty A_1 \Rightarrow nonempty (ty_2Epair_2Eprod \\ & A_0 A_1) \end{aligned} \quad (8)$$

Let $c_2Epair_2EAABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EAABS_prod \\ & A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (9)$$

Definition 29 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap (c_2$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow & \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ & A_27a A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod A_27a 2)^{A_27b}}) \end{aligned} \quad (10)$$

Definition 30 We define $c_2Epred_set_2EINTER$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2$

Definition 31 We define $c_2Epred_set_2EDISJOINT$ to be $\lambda A_27a : \iota. \lambda V0s \in (2^{A_27a}). \lambda V1t \in (2^{A_27a}). (ap (c_2$

Definition 32 We define $c_2Ebag_2EBAG_DISJOINT$ to be $\lambda A_27a : \iota. \lambda V0b1 \in (ty_2Enum_2Enum^{A_27a}).$

Definition 33 We define $c_2Ebag_2ESUB_BAG$ to be $\lambda A_27a : \iota. \lambda V0b1 \in (ty_2Enum_2Enum^{A_27a}). \lambda V1b1 \in (ty_2Enum_2Enum^{A_27a}). (ap (c_2$

Definition 34 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (\lambda V0x \in A_27a. (\lambda V1y \in A_27b. V0x = V1y))$

Definition 35 We define $c_2Ebag_2EEMPTY_BAG$ to be $\lambda A_27a : \iota. (ap (c_2Ecombin_2EK ty_2Enum_2Enum^{A_27a}))$

Definition 36 We define $c_2Ebag_2EFINITE_BAG$ to be $\lambda A_27a : \iota. \lambda V0b \in (ty_2Enum_2Enum^{A_27a}). (ap (c_2Ecombin_2EK ty_2Enum_2Enum^{A_27a}))$

Definition 37 We define c_2Ebag_2Emlt1 to be $\lambda A_27a : \iota. \lambda V0r \in ((2^{A_27a})^{A_27a}). \lambda V1b1 \in (ty_2Enum_2Enum^{A_27a}). (ap (c_2Ecombin_2EK ty_2Enum_2Enum^{A_27a}))$

Definition 38 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b \in A_27b. (V0R = V1a \wedge V0R = V2b)$

Definition 39 We define $c_2Erelation_2Etransitive$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap (c_2Ebool_2Bool^{A_27a}))$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (11)$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (12)$$

Definition 40 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap (ap (ap (c_2Ebool_2Bool^{A_27a}),$

Let $c_2Earithmetic_2EXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 41 We define $c_2Enumeral_2EiiSUC$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap c_2Enum_2ESUC (ap$

Definition 42 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 43 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 44 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 45 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota.\lambda V0R \in ((2^{A_27a})^{A_27a}).(ap (c_2Ebool_2E_21$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m)) \quad (15)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))) \\ & (16) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \\ & (17) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \\ & (18) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((\\ & \forall V2p \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B V0m) \\ & (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))) \\ & (19) \end{aligned}$$

Assume the following.

$$(p (ap (c_2Erelation_2Etransitive ty_2Enum_2Enum) c_2Eprim_rec_2E_3C)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n))) \quad (20)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC V0m)) V1n))))) \quad (21)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (22)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))) \quad (23)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D V0n) c_2Enum_2E0)) \Leftrightarrow (V0n = c_2Enum_2E0))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D V0m) c_2Enum_2E0) = V0m))) \quad (25)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D V0m) V1n) = c_2Enum_2E0) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n))))) \quad (26)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\
& (((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge \\
& ((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\
& (ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\
& V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\
& (ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\
& V0m) V1n)))))))))) \\
\end{aligned} \tag{27}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \\
\end{aligned} \tag{28}$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p (ap (ap c_2Earithmetic_2E_3C_3D \tag{29} \\
V0m) V0m)))$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& (p (ap (ap c_2Earithmetic_2E_3E_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n)))))) \\
\end{aligned} \tag{30}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& (V0m = V1n) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (\\
& ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m)))))) \\
\end{aligned} \tag{31}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. \\
& \forall V2p \in ty_2Enum_2Enum. ((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\
& V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \\
\end{aligned} \tag{32}$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & (ap c_2Enum_2ESUC V1n)) V0m)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (\neg(V0m = V1n)) \Leftrightarrow ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V0m)) V1n)) \vee (p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Enum_2ESUC \\ & V1n)) V0m))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0n) = (ap (ap \\ & c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)) V0n))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & \forall V2p \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D V0m) \\ & V1n) = V2p) \Leftrightarrow ((V0m = (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) \vee ((p \\ & (ap (ap c_2Earithmetic_2E_3C_3D V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D \\ & V2p) c_2Enum_2E0))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty_2Enum_2Enum}). (\forall V1a \in ty_2Enum_2Enum. \\ & (\forall V2b \in ty_2Enum_2Enum. ((p (ap V0P (ap (ap c_2Earithmetic_2E_2D \\ & V1a) V2b)) \Leftrightarrow (\forall V3d \in ty_2Enum_2Enum. (((V2b = (ap (ap c_2Earithmetic_2E_2B \\ & V1a) V3d)) \Rightarrow (p (ap V0P c_2Enum_2E0))) \wedge ((V1a = (ap (ap c_2Earithmetic_2E_2B \\ & V2b) V3d)) \Rightarrow (p (ap V0P V3d)))))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & ((V0b = (c_2Ebag_2EMPTY_BAG A_27a)) \vee (\exists V1b0 \in (ty_2Enum_2Enum^{A_27a}). \\ & (\exists V2e \in A_27a. (V0b = (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) \\ & V2e) V1b0))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & \forall A_27a. nonempty A_27a \Rightarrow (\forall V0b \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V1e1 \in A_27a. (\forall V2e2 \in A_27a. ((p (ap (ap (c_2Ebag_2EBAG_IN \\ & A_27a) V1e1) (ap (ap (c_2Ebag_2EBAG_INSERT A_27a) V2e2) V0b))) \Leftrightarrow \\ & ((V1e1 = V2e2) \vee (p (ap (ap (c_2Ebag_2EBAG_IN A_27a) V1e1) V0b))))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0x \in A_{.27a}.(\forall V1b \in \\ (ty_{.2Enum_{.2Enum^{A_{.27a}}}}).(\neg((ap\ (ap\ (c_{.2Ebag_{.2EBAG_INSERT}\ A_{.27a}}) \\ V0x)\ V1b) = (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27a}})))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\ nonempty\ A_{.27c} \Rightarrow & ((\forall V0b \in (ty_{.2Enum_{.2Enum^{A_{.27a}}}}).((ap\ (ap\ \\ (c_{.2Ebag_{.2EBAG_UNION}\ A_{.27a}})\ V0b)\ (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27a}})) = \\ V0b)) \wedge ((\forall V1b \in (ty_{.2Enum_{.2Enum^{A_{.27b}}}}).((ap\ (ap\ (c_{.2Ebag_{.2EBAG_UNION}\ \\ A_{.27b}})\ (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27b}}))\ V1b) = V1b))) \wedge (\forall V2b1 \in \\ (ty_{.2Enum_{.2Enum^{A_{.27c}}}}).(\forall V3b2 \in (ty_{.2Enum_{.2Enum^{A_{.27c}}}}). \\ (((ap\ (ap\ (c_{.2Ebag_{.2EBAG_UNION}\ A_{.27c}})\ V2b1)\ V3b2) = (c_{.2Ebag_{.2EMPTY_BAG}\ \\ A_{.27c}})) \Leftrightarrow ((V2b1 = (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27c}})) \wedge (V3b2 = (c_{.2Ebag_{.2EMPTY_BAG}\ \\ A_{.27c}})))))))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0x \in A_{.27a}.(\neg(p\ (ap\ (ap\ \\ (c_{.2Ebag_{.2EBAG_IN}\ A_{.27a}})\ V0x)\ (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27a}})))))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0b1 \in (ty_{.2Enum_{.2Enum^{A_{.27a}}}}). \\ (\forall V1b2 \in (ty_{.2Enum_{.2Enum^{A_{.27a}}}}).((p\ (ap\ (ap\ (c_{.2Ebag_{.2ESUB_BAG}\ \\ A_{.27a}})\ V0b1)\ V1b2)) \Leftrightarrow (\forall V2x \in A_{.27a}.(p\ (ap\ (ap\ c_{.2Earithmetic_{.2E_3C_3D}}\ \\ (ap\ V0b1\ V2x))\ (ap\ V1b2\ V2x))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\ nonempty\ A_{.27c} \Rightarrow & ((\forall V0b \in (ty_{.2Enum_{.2Enum^{A_{.27a}}}}).((ap\ (ap\ \\ (c_{.2Ebag_{.2EBAG_DIFF}\ A_{.27a}})\ V0b)\ (c_{.2Ebag_{.2EMPTY_BAG}\ \\ A_{.27a}}))) \wedge ((\forall V1b \in (ty_{.2Enum_{.2Enum^{A_{.27b}}}}).((ap\ (ap\ (c_{.2Ebag_{.2EBAG_DIFF}\ \\ A_{.27b}})\ (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27b}})) = V1b))) \wedge (\forall V2b \in \\ (ty_{.2Enum_{.2Enum^{A_{.27c}}}}).((ap\ (ap\ (c_{.2Ebag_{.2EBAG_DIFF}\ A_{.27c}})\ \\ (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27c}}))\ V2b) = (c_{.2Ebag_{.2EMPTY_BAG}\ A_{.27c}})))))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\forall V0x \in A_{.27a}.(\forall V1b1 \in \\ (ty_{.2Enum_{.2Enum^{A_{.27a}}}}).(\forall V2b2 \in (ty_{.2Enum_{.2Enum^{A_{.27a}}}}). \\ ((ap\ (ap\ (c_{.2Ebag_{.2EBAG_DIFF}\ A_{.27a}})\ (ap\ (ap\ (c_{.2Ebag_{.2EBAG_INSERT}\ \\ A_{.27a}})\ V0x)\ V1b1))\ (ap\ (ap\ (c_{.2Ebag_{.2EBAG_INSERT}\ A_{.27a}})\ V0x)\ V2b2)) = \\ (ap\ (ap\ (c_{.2Ebag_{.2EBAG_DIFF}\ A_{.27a}})\ V1b1)\ V2b2)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((\forall V0b1 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}). \\
& (\forall V1b2 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}).((p (ap (ap (c_{.2}Ebag_{.2}ESUB_{.2}BAG \\
& A_{.27a}) V0b1) V1b2)) \Rightarrow (\forall V2b3 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}).(p \\
& (ap (ap (c_{.2}Ebag_{.2}ESUB_{.2}BAG A_{.27a}) (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}DIFF \\
& A_{.27a}) V0b1) V2b3))) \wedge (\forall V3b1 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}). \\
& (\forall V4b2 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}).(\forall V5b3 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}). \\
& (\forall V6b4 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}).((p (ap (ap (c_{.2}Ebag_{.2}ESUB_{.2}BAG \\
& A_{.27a}) V4b2) V3b1)) \Rightarrow ((p (ap (ap (c_{.2}Ebag_{.2}ESUB_{.2}BAG A_{.27a}) V6b4) \\
& V5b3)) \Rightarrow ((p (ap (ap (c_{.2}Ebag_{.2}ESUB_{.2}BAG A_{.27a}) (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}DIFF \\
& A_{.27a}) V3b1) V4b2)) (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}DIFF A_{.27a}) V5b3) V6b4))) \Leftrightarrow \\
& (p (ap (ap (c_{.2}Ebag_{.2}ESUB_{.2}BAG A_{.27a}) (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}UNION \\
& A_{.27a}) V3b1) V6b4)) (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}UNION A_{.27a}) V4b2) \\
& V5b3)))))))))) \\
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((ap (c_{.2}Ebag_{.2}ESET_{.2}OF_{.2}BAG A_{.27a}) \\
& (c_{.2}Ebag_{.2}EEMPTY_{.2}BAG A_{.27a})) = (c_{.2}Epred_{.2}set_{.2}EEMPTY A_{.27a})) \\
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1b \in \\
& (ty_{.2}Enum_{.2}Enum^{A_{.27a}}).((p (ap (ap (c_{.2}Ebool_{.2}EIN A_{.27a}) V0x) \\
& (ap (c_{.2}Ebag_{.2}ESET_{.2}OF_{.2}BAG A_{.27a}) V1b))) \Leftrightarrow (p (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}IN \\
& A_{.27a}) V0x) V1b)))))) \\
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0b1 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}). \\
& (\forall V1b2 \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}).((p (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}DISJOINT \\
& A_{.27a}) V0b1) V1b2)) \Leftrightarrow (\forall V2e \in A_{.27a}.((\neg(p (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}IN \\
& A_{.27a}) V2e) V0b1))) \vee (\neg(p (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}IN A_{.27a}) V2e) \\
& V1b2))))))) \\
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((p (ap (c_{.2}Ebag_{.2}EFINITE_{.2}BAG \\
& A_{.27a}) (c_{.2}Ebag_{.2}EEMPTY_{.2}BAG A_{.27a}))) \wedge (\forall V0e \in A_{.27a}.(\\
& \forall V1b \in (ty_{.2}Enum_{.2}Enum^{A_{.27a}}).((p (ap (c_{.2}Ebag_{.2}EFINITE_{.2}BAG \\
& A_{.27a}) (ap (ap (c_{.2}Ebag_{.2}EBAG_{.2}INSERT A_{.27a}) V0e) V1b))) \Leftrightarrow (p (ap \\
& (c_{.2}Ebag_{.2}EFINITE_{.2}BAG A_{.27a}) V1b)))))) \\
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0R \in ((2^{A_{27a}})^{A_{27a}}). \\
& ((p (ap (c_2Erelation_2Etransitive A_{27a}) V0R)) \Rightarrow (\forall V1b1 \in \\
& (ty_2Enum_2Enum^{A_{27a}}).(\forall V2b2 \in (ty_2Enum_2Enum^{A_{27a}}) \\
& ((p (ap (ap (c_2Erelation_2ETC (ty_2Enum_2Enum^{A_{27a}})) (ap \\
& (c_2Ebag_2Emlt1 A_{27a}) V0R)) V1b1) V2b2)) \Leftrightarrow ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) V1b1)) \wedge ((p (ap (c_2Ebag_2EFINITE_BAG A_{27a}) V2b2)) \wedge \\
& \exists V3x \in (ty_2Enum_2Enum^{A_{27a}}).(\exists V4y \in (ty_2Enum_2Enum^{A_{27a}}) \\
& ((\neg(V3x = (c_2Ebag_2EEMPTY_BAG A_{27a}))) \wedge ((p (ap (ap (c_2Ebag_2ESUB_BAG \\
& A_{27a}) V3x) V2b2)) \wedge ((V1b1 = (ap (ap (c_2Ebag_2EBAG_UNION A_{27a}) \\
& (ap (ap (c_2Ebag_2EBAG_DIFF A_{27a}) V2b2) V3x) V4y)) \wedge (p (ap (ap \\
& (ap (c_2Ebag_2Edominates A_{27a} A_{27a}) V0R) (ap (c_2Ebag_2ESET_OF_BAG \\
& A_{27a}) V4y)) (ap (c_2Ebag_2ESET_OF_BAG A_{27a}) V3x))))))))))) \\
& (51)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0R \in ((2^{A_{27a}})^{A_{27a}}). \\
& (((p (ap (c_2Erelation_2EWF A_{27a}) V0R)) \wedge (p (ap (c_2Erelation_2Etransitive \\
& A_{27a}) V0R))) \Rightarrow (\forall V1b1 \in (ty_2Enum_2Enum^{A_{27a}}).(\forall V2b2 \in \\
& (ty_2Enum_2Enum^{A_{27a}}).((p (ap (ap (c_2Erelation_2ETC (ty_2Enum_2Enum^{A_{27a}})) \\
& (ap (c_2Ebag_2Emlt1 A_{27a}) V0R)) V1b1) V2b2)) \Leftrightarrow ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) V1b1)) \wedge ((p (ap (c_2Ebag_2EFINITE_BAG A_{27a}) V2b2)) \wedge \\
& \exists V3x \in (ty_2Enum_2Enum^{A_{27a}}).(\exists V4y \in (ty_2Enum_2Enum^{A_{27a}}) \\
& ((\neg(V3x = (c_2Ebag_2EEMPTY_BAG A_{27a}))) \wedge ((p (ap (ap (c_2Ebag_2ESUB_BAG \\
& A_{27a}) V3x) V2b2)) \wedge ((p (ap (ap (c_2Ebag_2EBAG_DISJOINT A_{27a}) \\
& V3x) V4y)) \wedge ((V1b1 = (ap (ap (c_2Ebag_2EBAG_UNION A_{27a}) (ap (ap \\
& (c_2Ebag_2EBAG_DIFF A_{27a}) V2b2) V3x) V4y)) \wedge (p (ap (ap (c_2Ebag_2Edominates \\
& A_{27a} A_{27a}) V0R) (ap (c_2Ebag_2ESET_OF_BAG A_{27a}) V4y)) (ap \\
& (c_2Ebag_2ESET_OF_BAG A_{27a}) V3x))))))))))) \\
& (52)
\end{aligned}$$

Assume the following.

$$True \quad (53)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\
& V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (56)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\
& A_{27a}.(p V0t)) \Leftrightarrow (p V0t)))
\end{aligned} \quad (57)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (60)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (61)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (62)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (V0x = V0x)) \quad (63)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (64)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (65)$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}). (\forall V1g \in (A_27b^{A_27a}). ((V0f = V1g) \Leftrightarrow (\forall V2x \in A_27a. ((ap V0f V2x) = (ap V1g V2x)))))) \quad (66)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (67)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ V0t1) V1t2) = V1t2)))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). (((p V0P) \wedge (\forall V2x \in A_27a. (p (ap V1Q V2x)))) \Leftrightarrow (\forall V3x \in \\ A_27a. ((p V0P) \wedge (p (ap V1Q V3x))))))) \end{aligned} \quad (69)$$

Assume the following.

$$\begin{aligned} \forall A_27a. & nonempty A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (\\ 2^{A_27a}). ((\forall V2x \in A_27a. ((p V0P) \vee (p (ap V1Q V2x)))) \Leftrightarrow ((p \\ V0P) \vee (\forall V3x \in A_27a. (p (ap V1Q V3x))))))) \end{aligned} \quad (70)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (\\ (p V1B) \vee (p V2C)))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C)))))) \quad (71)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(\\ p V0A)) \vee (\neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A)) \wedge (\neg(p V1B))))))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (\\ (p V1B) \wedge (p V2C)))) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (73)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee \\ (p V1B)))))) \quad (74)$$

Assume the following.

$$(\forall V0t \in 2. (((p V0t) \Rightarrow False) \Leftrightarrow ((p V0t) \Leftarrow False))) \quad (75)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (76)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p \\ V0t1) \Rightarrow (p V1t2)) \wedge ((p V1t2) \Rightarrow (p V0t1))))) \quad (77)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Leftrightarrow (p V1t2)) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \vee ((\neg(p V0t1) \wedge (\neg(p V1t2))))))) \quad (78)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in \\ & 2. (((p V0x) \Leftrightarrow (p V1x_27)) \wedge ((p V1x_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_27)))))) \Rightarrow \quad (79) \\ & (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_27) \Rightarrow (p V3y_27))))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap (ap (ap (c_2Ebool_2ECOND A_27a) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_2Ebool_2ECOND A_27a) V1Q) V3x_27) \\ & V5y_27)))))))) \quad (80) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0f \in (2^{A_27a}). (\forall V1v \in \\ & A_27a. ((\forall V2x \in A_27a. ((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (\quad (81) \\ & ap V0f V1v)))))) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) V2t1) V3t2) = V3t2)))))) \quad (82) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.\text{nonempty } A_27a \Rightarrow \forall A_27b.\text{nonempty } A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap (ap (c_2Ecombin_2EK \\ & A_27a A_27b) V0x) V1y) = V0x))) \quad (83) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (((ap c_2Enum_2ESUC c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)) \wedge ((\forall V0n \in ty_2Enum_2Enum. ((ap \\ & c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT1 V0n)) = (ap c_2Earithmetic_2EBIT2 \\ & V0n))) \wedge (\forall V1n \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC (ap c_2Earithmetic_2EBIT2 \\ & V1n)) = (ap c_2Earithmetic_2EBIT1 (ap c_2Enum_2ESUC V1n))))))) \quad (84) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enum_2EiZ (ap \\
& (ap c_2Earithmetic_2E_2B V2n) V3m))))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& ((\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
(ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
c_2Earithmetic_2EZERO)))) \wedge ((\forall V17n \in ty_2Enum_2Enum. \\
& (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Enum_2ESUC V17n)))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
(ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& ((\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
V30m) V29n)))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
V32n)))) \wedge ((\forall V33n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
c_2Enum_2E0) V33n)) \Leftrightarrow False)) \wedge ((\forall V34n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL \\
V34n)) \Leftrightarrow False)))))))
\end{aligned}$$

Assume the following.

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D c_2Earithmetic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& (ap c_2Earithmetic_2EBIT2 V0n)) c_2Earithmetic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT1 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT1 V1m))) \Leftrightarrow (\neg(p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V1m) V0n))) \wedge ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2EBIT2 \\
& V0n)) (ap c_2Earithmetic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D \\
& V0n) V1m))))))))))) \\
\end{aligned} \tag{87}$$

Assume the following.

$$\forall A_27a.\text{nonempty } A_27a \Rightarrow (\forall V0x \in A_27a. (\neg(p (ap (ap \\
c_2Ebool_2EIN A_27a) V0x) (c_2Epred_set_2EEMPTY A_27a))))) \tag{88}$$

Assume the following.

$$(p (ap (c_2Erelation_2EWF ty_2Enum_2Enum) c_2Eprim_rec_2E_3C)) \tag{89}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{90}$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{91}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \\
\end{aligned} \tag{93}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{94}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg \\
 & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
 & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
 & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge \\
 & ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
 & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow \\
 & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
 & ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \\
 \end{aligned} \tag{98}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
 (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \tag{99}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \tag{100}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{101}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))))) \tag{102}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))))) \tag{103}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{104}$$

Theorem 1

$$\begin{aligned}
 & (\forall V0b \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}).((p (ap (ap (ap \\
 & (c_2Erelation_2ETC (ty_2Enum_2Enum^{ty_2Enum_2Enum})) (ap (c_2Ebag_2Emlt1 \\
 & ty_2Enum_2Enum) c_2Eprim_rec_2E_3C)) (ap (ap (c_2Ebag_2EBAG_INSERT \\
 & ty_2Enum_2Enum) c_2Enum_2E0) (c_2Ebag_2EEMPTY_BAG ty_2Enum_2Enum)))) \\
 & V0b)) \Leftrightarrow ((p (ap (c_2Ebag_2EFINITE_BAG ty_2Enum_2Enum) V0b)) \wedge \\
 & ((\neg(V0b = (ap (ap (c_2Ebag_2EBAG_INSERT ty_2Enum_2Enum) c_2Enum_2E0) \\
 & (c_2Ebag_2EEMPTY_BAG ty_2Enum_2Enum)))) \wedge (\neg(V0b = (c_2Ebag_2EEMPTY_BAG \\
 & ty_2Enum_2Enum)))))))
 \end{aligned}$$