

thm\_2Ebag\_2Emlt\_\_UNION\_\_EMPTY\_\_EQN  
 (TMFtz9zNKfmA6k3zVKxs9uZzA1fH6K9CENm)

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Let  $c\_2Enum\_2ZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ZERO\_REP \in \omega \quad (1)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let  $c\_2Enum\_2ABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2ABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (3)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be ( $ap\ c\_2Enum\_2ABS\_num\ c\_2Enum\_2ZERO\_REP$ ).

**Definition 3** We define  $c\_2Earithmetic\_2ZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be ( $ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x)$ )

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x)))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2ABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$



Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b1 \in (ty\_2Enum\_2Enum^{A_{27a}}). \\ & (\forall V1b2 \in (ty\_2Enum\_2Enum^{A_{27a}}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION \\ A_{27a})\ V0b1)\ V1b2) = (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A_{27a})\ V1b2) \\ & V0b1)))))) \end{aligned} \quad (7)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & \forall A_{27b}.nonempty\ A_{27b} \Rightarrow \forall A_{27c}. \\ & nonempty\ A_{27c} \Rightarrow ((\forall V0b \in (ty\_2Enum\_2Enum^{A_{27a}}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A_{27a})\ V0b) \\ (c\_2Ebag\_2EEMPTY\_BAG\ A_{27a})) = V0b)) \wedge ((\forall V1b \in (ty\_2Enum\_2Enum^{A_{27b}}).((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A_{27b})\ V1b) \\ (c\_2Ebag\_2EEMPTY\_BAG\ A_{27b})) = V1b)) \wedge (\forall V2b1 \in \\ & (ty\_2Enum\_2Enum^{A_{27c}}).(\forall V3b2 \in (ty\_2Enum\_2Enum^{A_{27c}}). \\ & (((ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A_{27c})\ V2b1)\ V3b2) = (c\_2Ebag\_2EEMPTY\_BAG\ A_{27c})) \Leftrightarrow ((V2b1 = (c\_2Ebag\_2EEMPTY\_BAG\ A_{27c})) \wedge (V3b2 = (c\_2Ebag\_2EEMPTY\_BAG\ A_{27c})))))))))) \end{aligned} \quad (8)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & ((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ (c\_2Ebag\_2EEMPTY\_BAG\ A_{27a}))) \wedge (\forall V0e \in A_{27a}.( \\ & \forall V1b \in (ty\_2Enum\_2Enum^{A_{27a}}).((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_INSERT\ A_{27a})\ V0e)\ V1b))) \Leftrightarrow (p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ V1b))))))) \end{aligned} \quad (9)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0b1 \in (ty\_2Enum\_2Enum^{A_{27a}}). \\ & (\forall V1b2 \in (ty\_2Enum\_2Enum^{A_{27a}}).((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ (ap\ (ap\ (c\_2Ebag\_2EBAG\_UNION\ A_{27a})\ V0b1)\ V1b2))) \Leftrightarrow ((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ V0b1)) \wedge (p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ V1b2))))))) \end{aligned} \quad (10)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0R \in ((2^{A_{27a}})^{A_{27a}}). \\ & ((p\ (ap\ (c\_2Erelation\_2EWF\ A_{27a})\ V0R)) \Rightarrow (p\ (ap\ (c\_2Erelation\_2EWF\ (ty\_2Enum\_2Enum^{A_{27a}}))\ (ap\ (c\_2Ebag\_2Emlt1\ A_{27a})\ V0R)))))) \end{aligned} \quad (11)$$

Assume the following.

$$\begin{aligned} \forall A_{27a}.nonempty\ A_{27a} \Rightarrow & (\forall V0R \in ((2^{A_{27a}})^{A_{27a}}). \\ & (\forall V1b1 \in (ty\_2Enum\_2Enum^{A_{27a}}).(\forall V2b2 \in (ty\_2Enum\_2Enum^{A_{27a}}). \\ & ((p\ (ap\ (ap\ (c\_2Erelation\_2ETC\ (ty\_2Enum\_2Enum^{A_{27a}}))\ (ap\ (c\_2Ebag\_2Emlt1\ A_{27a})\ V0R))\ V1b1)\ V2b2)) \Rightarrow ((p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ V1b1)) \wedge (p\ (ap\ (c\_2Ebag\_2EFINITE\_BAG\ A_{27a})\ V2b2))))))) \end{aligned} \quad (12)$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0R \in ((2^{A_{27a}})^{A_{27a}}). \\
& (\forall V1b2 \in (ty\_2Enum\_2Enum^{A_{27a}}). (\forall V2b1 \in (ty\_2Enum\_2Enum^{A_{27a}}). \\
& (((p (ap (c_2Ebag_2EFINITE_BAG A_{27a}) V1b2)) \wedge ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) V2b1)) \wedge (\neg(V1b2 = (c_2Ebag_2EEMPTY_BAG A_{27a})))))) \Rightarrow (p \\
& (ap (ap (ap (c_2Erelation_2ETC (ty\_2Enum\_2Enum^{A_{27a}})) (ap (c_2Ebag_2Emlt1 \\
& A_{27a}) V0R) V2b1) (ap (ap (c_2Ebag_2EBAG_UNION A_{27a}) V2b1) V1b2)))))) \\
& (13)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0R \in ((2^{A_{27a}})^{A_{27a}}). \\
& (\forall V1b1 \in (ty\_2Enum\_2Enum^{A_{27a}}). (\forall V2b2 \in (ty\_2Enum\_2Enum^{A_{27a}}). \\
& ((p (ap (c_2Erelation_2EWF A_{27a}) V0R)) \Rightarrow (((p (ap (ap (ap (c_2Erelation_2ETC \\
& (ty\_2Enum\_2Enum^{A_{27a}})) (ap (c_2Ebag_2Emlt1 A_{27a}) V0R)) V1b1) \\
& (ap (ap (c_2Ebag_2EBAG_UNION A_{27a}) V1b1) V2b2))) \Leftrightarrow ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) V1b1)) \wedge ((p (ap (c_2Ebag_2EFINITE_BAG A_{27a}) V2b2)) \wedge ( \\
& \neg(V2b2 = (c_2Ebag_2EEMPTY_BAG A_{27a})))))) \wedge ((p (ap (ap (c_2Erelation_2ETC \\
& (ty\_2Enum\_2Enum^{A_{27a}})) (ap (c_2Ebag_2Emlt1 A_{27a}) V0R)) V1b1) \\
& (ap (ap (c_2Ebag_2EBAG_UNION A_{27a}) V2b2) V1b1)) \Leftrightarrow ((p (ap (c_2Ebag_2EFINITE_BAG \\
& A_{27a}) V1b1)) \wedge ((p (ap (c_2Ebag_2EFINITE_BAG A_{27a}) V2b2)) \wedge ( \\
& \neg(V2b2 = (c_2Ebag_2EEMPTY_BAG A_{27a})))))))))) \\
& (14)
\end{aligned}$$

Assume the following.

$$True \quad (15)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (17)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \\
& (18)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))) \\
& (19)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2. ((\neg(\neg(p V0t)) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))))) \\
& (20)
\end{aligned}$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (V0x = V0x)) \quad (21)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow \text{True})) \quad (22)$$

Assume the following.

$$\forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2. (((\text{True} \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow \text{True}) \Leftrightarrow (p V0t)) \wedge (((\text{False} \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow \text{False}) \Leftrightarrow (\neg(p V0t))))))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V0A) \vee (p V1B) \wedge (p V2C)) \Leftrightarrow (((p V0A) \vee (p V1B)) \wedge ((p V0A) \vee (p V2C))))))) \quad (25)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p V1B) \wedge (p V2C)) \vee (p V0A)) \Leftrightarrow (((p V1B) \vee (p V0A)) \wedge ((p V2C) \vee (p V0A))))))) \quad (26)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (27)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((p V0x) \Leftrightarrow (p V1x\_27)) \wedge ((p V1x\_27) \Rightarrow ((p V2y) \Leftrightarrow (p V3y\_27)))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x\_27) \Rightarrow (p V3y\_27))))))) \quad (28)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & (\forall V1x \in A\_27a. (\forall V2y \in A\_27a. ((p (ap (\text{c\_2Erelation\_2EWF}} \\ & A\_27a) V0R)) \Rightarrow ((p (ap (ap V0R V1x) V2y)) \Rightarrow (\neg(V1x = V2y))))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.\text{nonempty } A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). \\ & ((p (ap (\text{c\_2Erelation\_2EWF}} A\_27a) (ap (\text{c\_2Erelation\_2ETC}} A\_27a) \\ & V0R)) \Leftrightarrow (p (ap (\text{c\_2Erelation\_2EWF}} A\_27a) V0R))) \end{aligned} \quad (30)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (32)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (33)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (34)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & ((\neg(p V0p)) \wedge ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \wedge ((p V0p) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow \\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))))))) \end{aligned} \quad (39)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (45)$$

**Theorem 1** *True.*