thm_2Ebag_2Emlt__UNION__LCANCEL (TMM-PAbBX46R93Xj4etJEKHKtm3HateTx5ox)

October 26, 2020

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (1)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (2)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (3)$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 3 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (5)$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 6 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Definition 7 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 8 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t)).$

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$ *of type* ι.

Definition 11 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge$ *of type* $\iota{\Rightarrow}\iota$.

Definition 13 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 14 *We define* c_2Ebag_2EBAG__INSERT *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2E$

Definition 15 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 16 *We define* c_2Ebag_2EEMPTY__BAG *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Ecombin\_2EK}\ ty\_2Enum\_2En$

Definition 17 *We define* c_2Ebag_2EFINITE__BAG *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).(ap$

Definition 18 *We define* c_2Ebag_2EBAG__UNION *to be* $\lambda A\_27a : \iota.\lambda V0b \in (ty\_2Enum\_2Enum^{A\_27a}).\lambda V1$

Definition 19 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 20 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 21 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 22 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 23 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 24 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 25 *We define* c_2Ebag_2EBAG__INN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1n \in ty\_2Enum\_2Enum$

Definition 26 *We define* c_2Ebag_2EBAG__IN *to be* $\lambda A\_27a : \iota.\lambda V0e \in A\_27a.\lambda V1b \in (ty\_2Enum\_2Enum$

Definition 27 *We define* c_2Ebag_2Emlt1 *to be* $\lambda A\_27a : \iota.\lambda V0r \in ((2^{A\_27a})^{A\_27a}).\lambda V1b1 \in (ty\_2Enum\_2E$

Definition 28 *We define* c_2Erelation_2ETC *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).\lambda V1a \in A\_27a.\lambda V2b$

Definition 29 *We define* c_2Erelation_2Etransitive *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2}$

Definition 30 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b1 \in (ty\_2Enum\_2Enum^{A\_27a}). (\forall V1b2 \in (ty\_2Enum\_2Enum^{A\_27a}).((ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V0b1)\ V1b2) = (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1b2)\ V0b1)))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1a \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2c \in (ty\_2Enum\_2Enum^{A\_27a}). (\forall V3b \in (ty\_2Enum\_2Enum^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0R)) \wedge (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27a)\ V0R))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Enum\_2Enum^{A\_27a}))\ (ap\ (\mathsf{c\_2Ebag\_2Emlt1}\ A\_27a)\ V0R))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1a)\ V2c))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V3b)\ V2c))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Enum\_2Enum^{A\_27a}))\ (ap\ (\mathsf{c\_2Ebag\_2Emlt1}\ A\_27a)\ V0R))\ V1a)\ V3b)) \wedge (p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V2c))))))))) \tag{8}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0R \in ((2^{A\_27a})^{A\_27a}). (\forall V1c \in (ty\_2Enum\_2Enum^{A\_27a}).(\forall V2a \in (ty\_2Enum\_2Enum^{A\_27a}). (\forall V3b \in (ty\_2Enum\_2Enum^{A\_27a}).(((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0R)) \wedge (p\ (ap\ (\mathsf{c\_2Erelation\_2Etransitive}\ A\_27a)\ V0R))) \Rightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Enum\_2Enum^{A\_27a}))\ (ap\ (\mathsf{c\_2Ebag\_2Emlt1}\ A\_27a)\ V0R))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1c)\ V2a))\ (ap\ (ap\ (\mathsf{c\_2Ebag\_2EBAG\_\_UNION}\ A\_27a)\ V1c)\ V3b))) \Leftrightarrow ((p\ (ap\ (ap\ (ap\ (\mathsf{c\_2Erelation\_2ETC}\ (ty\_2Enum\_2Enum^{A\_27a}))\ (ap\ (\mathsf{c\_2Ebag\_2Emlt1}\ A\_27a)\ V0R))\ V2a)\ V3b)) \wedge (p\ (ap\ (\mathsf{c\_2Ebag\_2EFINITE\_\_BAG}\ A\_27a)\ V1c)))))))))$$