

thm_2Ebag_2Emlt__UNION__RCANCEL__I
(TMcQcVkzmST4P6nJSbBvN9qDjipLzKbVT22)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \tag{2}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 4 We define $c_2Ebag_2EBAG_UNION$ to be $\lambda A_27a : \iota.\lambda V0b \in (ty_2Enum_2Enum^{A_27a}).\lambda V1c$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 5 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 6 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{5}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{6}$$

- Definition 7** We define `c_2Enum_2ESUC` to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ ($
- Definition 8** We define `c_2Arithmetic_2EBIT1` to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Arithmetic_2E$
- Definition 9** We define `c_2Arithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x.$
- Definition 10** We define `c_2Ebool_2EF` to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t)).$
- Definition 11** We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$
of type ι .
- Definition 12** We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V0t2))$
- Definition 13** We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x))$
of type $\iota \Rightarrow \iota$.
- Definition 14** We define `c_2Ebool_2ECOND` to be $\lambda A.\lambda a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.\lambda a.\lambda V2t2 \in A.\lambda a.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t2 \in 2.V0t2))$
- Definition 15** We define `c_2Ebag_2EBAG_INSERT` to be $\lambda A.\lambda a : \iota.\lambda V0e \in A.\lambda a.\lambda V1b \in (ty_2Enum_2Enum\ A).$
- Definition 16** We define `c_2Ecombin_2EK` to be $\lambda A.\lambda a : \iota.\lambda A.\lambda a.\lambda V0x \in A.\lambda a.\lambda V1y \in A.\lambda a.V0x$
- Definition 17** We define `c_2Ebag_2EEMPTY_BAG` to be $\lambda A.\lambda a : \iota.(ap\ (c_2Ecombin_2EK\ ty_2Enum_2Enum\ A))$
- Definition 18** We define `c_2Ebag_2EFINITE_BAG` to be $\lambda A.\lambda a : \iota.\lambda V0b \in (ty_2Enum_2Enum\ A^{27a}).(ap\ (c_2Ecombin_2EK\ ty_2Enum_2Enum\ A^{27a}))$
- Definition 19** We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$
- Definition 20** We define `c_2Ebool_2E_3F` to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ V0P))$
- Definition 21** We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum\ A.$
- Definition 22** We define `c_2Arithmetic_2E_3E` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum\ A.$
- Definition 23** We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2.V0t2))$
- Definition 24** We define `c_2Arithmetic_2E_3E_3D` to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum\ A.$
- Definition 25** We define `c_2Ebag_2EBAG_INN` to be $\lambda A.\lambda a : \iota.\lambda V0e \in A.\lambda a.\lambda V1n \in ty_2Enum_2Enum\ A.$
- Definition 26** We define `c_2Ebag_2EBAG_IN` to be $\lambda A.\lambda a : \iota.\lambda V0e \in A.\lambda a.\lambda V1b \in (ty_2Enum_2Enum\ A^{27a}).$
- Definition 27** We define `c_2Ebag_2Emlt1` to be $\lambda A.\lambda a : \iota.\lambda V0r \in ((2^{A-27a})^{A-27a}).\lambda V1b1 \in (ty_2Enum_2Enum\ A^{27a}).$
- Definition 28** We define `c_2ERelation_2ETC` to be $\lambda A.\lambda a : \iota.\lambda V0R \in ((2^{A-27a})^{A-27a}).\lambda V1a \in A.\lambda V2b \in A.$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0e \in A.27a. (\forall V1b1 \in \\
& \quad (ty_2Enum_2Enum^{A.27a}). (\forall V2b2 \in (ty_2Enum_2Enum^{A.27a}). \\
& \quad (((ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A.27a)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT \\
& \quad A.27a)\ V0e)\ V1b1))\ V2b2) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A.27a) \\
& \quad V0e)\ (ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A.27a)\ V1b1)\ V2b2)))) \wedge ((ap\ (\\
& \quad ap\ (c_2Ebag_2EBAG_UNION\ A.27a)\ V1b1)\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT \\
& \quad A.27a)\ V0e)\ V2b2)) = (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A.27a)\ V0e) \\
& \quad (ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A.27a)\ V1b1)\ V2b2))))))
\end{aligned} \tag{7}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow ((\forall V0b \in (ty_2Enum_2Enum^{A.27a}). ((ap\ (ap \\
& \quad (c_2Ebag_2EBAG_UNION\ A.27a)\ V0b)\ (c_2Ebag_2EEMPTY_BAG\ A.27a)) = \\
& \quad V0b)) \wedge ((\forall V1b \in (ty_2Enum_2Enum^{A.27b}). ((ap\ (ap\ (c_2Ebag_2EBAG_UNION \\
& \quad A.27b)\ (c_2Ebag_2EEMPTY_BAG\ A.27b))\ V1b) = V1b)) \wedge (\forall V2b1 \in \\
& \quad (ty_2Enum_2Enum^{A.27c}). (\forall V3b2 \in (ty_2Enum_2Enum^{A.27c}). \\
& \quad (((ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A.27c)\ V2b1)\ V3b2) = (c_2Ebag_2EEMPTY_BAG \\
& \quad A.27c)) \Leftrightarrow ((V2b1 = (c_2Ebag_2EEMPTY_BAG\ A.27c)) \wedge (V3b2 = (c_2Ebag_2EEMPTY_BAG \\
& \quad A.27c))))))
\end{aligned} \tag{8}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Enum_2Enum^{A.27a})}). \\
& \quad (((p\ (ap\ V0P\ (c_2Ebag_2EEMPTY_BAG\ A.27a))) \wedge (\forall V1b \in (ty_2Enum_2Enum^{A.27a}). \\
& \quad (((p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A.27a)\ V1b)) \wedge (p\ (ap\ V0P\ V1b)))) \Rightarrow \\
& \quad (\forall V2e \in A.27a. (p\ (ap\ V0P\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT\ A.27a) \\
& \quad V2e)\ V1b)))))) \Rightarrow (\forall V3b \in (ty_2Enum_2Enum^{A.27a}). ((p\ (ap \\
& \quad (c_2Ebag_2EFINITE_BAG\ A.27a)\ V3b)) \Rightarrow (p\ (ap\ V0P\ V3b))))))
\end{aligned} \tag{9}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0R \in ((2^{A.27a})^{A.27a}). \\
& \quad (\forall V1e \in A.27a. (\forall V2a \in (ty_2Enum_2Enum^{A.27a}). (\forall V3b \in \\
& \quad (ty_2Enum_2Enum^{A.27a}). ((p\ (ap\ (ap\ (ap\ (c_2Erelation_2ETC\ (ty_2Enum_2Enum^{A.27a})) \\
& \quad (ap\ (c_2Ebag_2Emlt1\ A.27a)\ V0R))\ V2a)\ V3b)) \Rightarrow (p\ (ap\ (ap\ (ap\ (c_2Erelation_2ETC \\
& \quad (ty_2Enum_2Enum^{A.27a}))\ (ap\ (c_2Ebag_2Emlt1\ A.27a)\ V0R))\ (ap\ (\\
& \quad ap\ (c_2Ebag_2EBAG_INSERT\ A.27a)\ V1e)\ V2a))\ (ap\ (ap\ (c_2Ebag_2EBAG_INSERT \\
& \quad A.27a)\ V1e)\ V3b))))))
\end{aligned} \tag{10}$$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A.27a. (p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{12}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (14)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (15)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (16)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (17)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0Q \in 2.(\forall V1P \in (2^{A.27a}).((\forall V2x \in A.27a.((p (ap V1P V2x)) \vee (p V0Q))) \Leftrightarrow ((\forall V3x \in A.27a.(p (ap V1P V3x))) \vee (p V0Q)))))) \quad (18)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (19)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow (((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (23)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg((p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (24)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (25)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (26)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (27)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (28)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (29)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (30)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (31)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\ & (\forall V1a \in (ty_2Enum_2Enum^{A_27a}).(\forall V2b \in (ty_2Enum_2Enum^{A_27a}). \\ & (\forall V3c \in (ty_2Enum_2Enum^{A_27a}).(((p\ (ap\ (ap\ (ap\ (c_2Erelation_2ETC \\ & (ty_2Enum_2Enum^{A_27a}))\ (ap\ (c_2Ebag_2Emlt1\ A_27a)\ V0R))\ V1a) \\ & V2b)) \wedge (p\ (ap\ (c_2Ebag_2EFINITE_BAG\ A_27a)\ V3c))) \Rightarrow (p\ (ap\ (ap\ (\\ & ap\ (c_2Erelation_2ETC\ (ty_2Enum_2Enum^{A_27a}))\ (ap\ (c_2Ebag_2Emlt1 \\ & A_27a)\ V0R))\ (ap\ (ap\ (c_2Ebag_2EBAG_UNION\ A_27a)\ V1a)\ V3c))\ (ap \\ & (ap\ (c_2Ebag_2EBAG_UNION\ A_27a)\ V2b)\ V3c)))))) \end{aligned}$$