

# thm\_2Ebft\_2EBFT\_FOLD

(TMNobrbz3FZsVxvCtTEC54m6e3M6L9AvCs9)

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Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (1)$$

Let  $c\_2Elist\_2ELIST\_TO\_SET : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ELIST\_TO\_SET\ A\_27a \in ((2^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27a)}) \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Let  $c\_2Ebft\_2EBFT : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Ebft\_2EBFT\ A\_27a\ A\_27b \in (((((A\_27a^{A\_27a})^{(ty\_2Elist\_2Elist\ A\_27b)})^{(ty\_2Elist\_2Elist\ A\_27b)})^{((A\_27a^{A\_27a})^{A\_27b})})^{(ty\_2Elist\_2Elist\ A\_27b)}) \quad (3)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (4)$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1Q \in 2.V1Q))\ (\lambda V2R \in 2.V2R)))$

**Definition 5** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 6** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Epair\_2Eprod A0 A1) \quad (5)$$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b}})^{A\_27a}) \quad (6)$$

**Definition 9** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC A\_27a A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod A\_27a 2)^{A\_27b}}) \quad (7)$$

**Definition 10** We define  $c\_2EdirGraph\_2EParents$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0G \in ((ty\_2Elist\_2Elist A$

**Definition 11** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.$

**Definition 12** We define  $c\_2Epred\_set\_2EINSERT$  to be  $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1s \in (2^{A\_27a}).(ap (c\_2$

**Definition 13** We define  $c\_2Epred\_set\_2EEMPTY$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.c\_2Ebool\_2EF).$

**Definition 14** We define  $c\_2Epred\_set\_2EFINITE$  to be  $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).(ap (c\_2Ebool\_2E\_21 2)$

**Definition 15** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$   
of type  $\iota \Rightarrow \iota$ .

**Definition 16** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(\lambda V2t3 \in 2.$

Let  $c\_2Elist\_2EAPPEND : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2EAPPEND A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{(ty\_2Elist\_2Elist A\_27a)}) \quad (8)$$

Let  $c\_2Elist\_2EFOLDR : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Elist\_2EFOLDR A\_27a A\_27b \in (((A\_27b^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27b})^{(A\_27b^{A\_27b})^{A\_27a}}) \quad (9)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Elist\_2ECONS A\_27a \in (((ty\_2Elist\_2Elist A\_27a)^{(ty\_2Elist\_2Elist A\_27a)})^{A\_27a}) \quad (10)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0G \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27a}). (\forall V1f \in ( \\
& \quad (A\_27b^{A\_27b})^{A\_27a}). (\forall V2seen \in (ty\_2Elist\_2Elist\ A\_27a). \\
& \quad (\forall V3acc \in A\_27b. (\forall V4h \in A\_27a. (\forall V5t \in (ty\_2Elist\_2Elist \\
& \quad A\_27a). ((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a)\ (ap\ (c\_2EdirGraph\_2EParents \\
& \quad A\_27a\ A\_27a)\ V0G))) \Rightarrow (((ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ebft\_2EBFT\ A\_27b\ A\_27a) \\
& \quad V0G)\ V1f)\ V2seen)\ (c\_2Elist\_2ENIL\ A\_27a))\ V3acc) = V3acc) \wedge ((ap \\
& \quad (ap\ (ap\ (ap\ (ap\ (c\_2Ebft\_2EBFT\ A\_27b\ A\_27a)\ V0G)\ V1f)\ V2seen)\ (ap \\
& \quad (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4h)\ V5t))\ V3acc) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND \\
& \quad A\_27b)\ (ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V4h)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET \\
& \quad A\_27a)\ V2seen)))\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ebft\_2EBFT\ A\_27b\ A\_27a) \\
& \quad V0G)\ V1f)\ V2seen)\ V5t)\ V3acc))\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2Ebft\_2EBFT \\
& \quad A\_27b\ A\_27a)\ V0G)\ V1f)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V4h)\ V2seen)) \\
& \quad (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V5t)\ (ap\ V0G\ V4h)))\ (ap\ (ap\ V1f \\
& \quad V4h)\ V3acc))))))))) \\
& \hspace{15em} (11)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0P \in (((((2^{A\_27b})(ty\_2Elist\_2Elist\ A\_27a))(ty\_2Elist\_2Elist\ A\_27a))^{(A\_27b^{A\_27b})^{A\_27a}}))^{(ty\_2Elist\_2Elist\ A\_27a)^{A\_27a}} \\
& \quad ((\forall V1G \in ((ty\_2Elist\_2Elist\ A\_27a)^{A\_27a}). (\forall V2f \in \\
& \quad ((A\_27b^{A\_27b})^{A\_27a}). (\forall V3seen \in (ty\_2Elist\_2Elist\ A\_27a). \\
& \quad (\forall V4h \in A\_27a. (\forall V5t \in (ty\_2Elist\_2Elist\ A\_27a). ( \\
& \quad \forall V6acc \in A\_27b. ((p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ V3seen) \\
& \quad (c\_2Elist\_2ENIL\ A\_27a))\ V6acc) \wedge (((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\
& \quad A\_27a)\ (ap\ (c\_2EdirGraph\_2EParents\ A\_27a\ A\_27a)\ V1G))) \wedge (\neg(p\ ( \\
& \quad ap\ (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V4h)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET \\
& \quad A\_27a)\ V3seen)))))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad A\_27a)\ V4h)\ V3seen))\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V5t)\ (ap \\
& \quad V1G\ V4h)))\ (ap\ (ap\ V2f\ V4h)\ V6acc)))) \wedge (((p\ (ap\ (c\_2Epred\_set\_2EFINITE \\
& \quad A\_27a)\ (ap\ (c\_2EdirGraph\_2EParents\ A\_27a\ A\_27a)\ V1G))) \wedge (p\ (ap \\
& \quad (ap\ (c\_2Ebool\_2EIN\ A\_27a)\ V4h)\ (ap\ (c\_2Elist\_2ELIST\_TO\_SET \\
& \quad A\_27a)\ V3seen)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ V3seen)\ V5t) \\
& \quad V6acc)))) \Rightarrow (p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V1G)\ V2f)\ V3seen)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\
& \quad A\_27a)\ V4h)\ V5t))\ V6acc)))))) \Rightarrow (\forall V7v \in ((ty\_2Elist\_2Elist \\
& \quad A\_27a)^{A\_27a}). (\forall V8v1 \in ((A\_27b^{A\_27b})^{A\_27a}). (\forall V9v2 \in \\
& \quad (ty\_2Elist\_2Elist\ A\_27a). (\forall V10v3 \in (ty\_2Elist\_2Elist \\
& \quad A\_27a). (\forall V11v4 \in A\_27b. (p\ (ap\ (ap\ (ap\ (ap\ (ap\ V0P\ V7v)\ V8v1) \\
& \quad V9v2)\ V10v3)\ V11v4))))))))) \\
& \hspace{15em} (12)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0G \in ((ty\_2Elist\_2Elist \\
& A\_27a)^{A\_27a}).(\forall V1f \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}). \\
& (\forall V2seen \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3fringe \in \\
& (ty\_2Elist\_2Elist\ A\_27a).(\forall V4acc \in (ty\_2Elist\_2Elist \\
& A\_27a).(\forall V5a \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V6b \in \\
& (ty\_2Elist\_2Elist\ A\_27a).(((p\ (ap\ (c\_2Epred\_set\_2EFINITE\ A\_27a) \\
& (ap\ (c\_2EdirGraph\_2EParents\ A\_27a\ A\_27a)\ V0G))) \wedge ((V1f = (c\_2Elist\_2ECONS \\
& A\_27a)) \wedge (V4acc = (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V5a)\ V6b)))))) \Rightarrow \\
& ((ap\ (ap\ (ap\ (ap\ (ap\ (c\_2EbfT\_2EBFT\ (ty\_2Elist\_2Elist\ A\_27a)\ A\_27a) \\
& V0G)\ V1f)\ V2seen)\ V3fringe)\ V4acc) = (ap\ (ap\ (c\_2Elist\_2EAPPEND \\
& A\_27a)\ (ap\ (ap\ (ap\ (ap\ (ap\ (c\_2EbfT\_2EBFT\ (ty\_2Elist\_2Elist\ A\_27a) \\
& A\_27a)\ V0G)\ V1f)\ V2seen)\ V3fringe)\ V5a))\ V6b)))))))))
\end{aligned} \tag{13}$$

Assume the following.

$$True \tag{14}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{16}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \vee (\neg(p\ V0t)))) \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow \\
& (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))
\end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\
& (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))
\end{aligned} \tag{19}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
& ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(V0x = V0x)) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \end{aligned} \quad (24)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}). (\forall V1Q \in \\ & (2^{A\_27a}). ((\forall V2x \in A\_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow \\ & ((\forall V3x \in A\_27a. (p\ (ap\ V0P\ V3x))) \wedge (\forall V4x \in A\_27a. (p\ ( \\ & ap\ V1Q\ V4x)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in ( \\ & 2^{A\_27a}). (((p\ V0P) \wedge (\forall V2x \in A\_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in \\ & A\_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & V5y\_27)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a. (\forall V3t2 \in A\_27a. ((ap \\ & (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF)\ V2t1)\ V3t2) = V3t2)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0l \in (ty\_2Elist\_2Elist \\ & A\_27a). ((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a)) \\ & V0l) = V0l)) \wedge (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a). (\forall V2l2 \in \\ & (ty\_2Elist\_2Elist\ A\_27a). (\forall V3h \in A\_27a. ((ap\ (ap\ (c\_2Elist\_2EAPPEND \\ & A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap \\ & (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a) \\ & V1l1)\ V2l2)))))))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow & ( \\ & (\forall V0f \in ((A\_27b^{A\_27b})^{A\_27a}). (\forall V1e \in A\_27b. ((ap\ ( \\ & ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V0f)\ V1e)\ (c\_2Elist\_2ENIL \\ & A\_27a)) = V1e))) \wedge (\forall V2f \in ((A\_27b^{A\_27b})^{A\_27a}). (\forall V3e \in \\ & A\_27b. (\forall V4x \in A\_27a. (\forall V5l \in (ty\_2Elist\_2Elist\ A\_27a). \\ & ((ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR\ A\_27a\ A\_27b)\ V2f)\ V3e)\ (ap\ (ap\ (c\_2Elist\_2ECONS \\ & A\_27a)\ V4x)\ V5l)) = (ap\ (ap\ V2f\ V4x)\ (ap\ (ap\ (ap\ (c\_2Elist\_2EFOLDR \\ & A\_27a\ A\_27b)\ V2f)\ V3e)\ V5l)))))))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). \\ & (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist \\ & A\_27a). ((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a. (p\ (ap\ V0P\ (ap\ (ap\ ( \\ & c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t)))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist \\ & A\_27a). (p\ (ap\ V0P\ V3l)))))) \end{aligned} \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (37)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (38)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (39)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (40)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (41)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \vee (p V2r)) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((p V1q) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (42)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (43)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (44)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (45)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \quad (46)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \quad (47)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p V0p) \vee (p V1q))) \Rightarrow \neg(p V1q)))) \quad (48)$$

Assume the following.

$$(\forall V0p \in 2.((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \quad (49)$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty \ A\_27a \Rightarrow \forall A\_27b.nonempty \ A\_27b \Rightarrow ( \\ & \quad \forall V0G \in ((ty\_2Elist\_2Elist \ A\_27a)^{A\_27a}).(\forall V1f \in ( \\ & \quad (A\_27b^{A\_27b})^{A\_27a}).(\forall V2seen \in (ty\_2Elist\_2Elist \ A\_27a). \\ & \quad (\forall V3fringe \in (ty\_2Elist\_2Elist \ A\_27a).(\forall V4acc \in \\ & \quad A\_27b.((p \ (ap \ (c\_2Epred\_set\_2EFINITE \ A\_27a) \ (ap \ (c\_2EdirGraph\_2EParents \\ & \quad A\_27a \ A\_27a) \ V0G))) \Rightarrow ((ap \ (ap \ (ap \ (ap \ (ap \ (c\_2Ebft\_2EBFT \ A\_27b \ A\_27a) \\ & \quad V0G) \ V1f) \ V2seen) \ V3fringe) \ V4acc) = (ap \ (ap \ (ap \ (c\_2Elist\_2EFOLDR \\ & \quad A\_27a \ A\_27b) \ V1f) \ V4acc) \ (ap \ (ap \ (ap \ (ap \ (c\_2Ebft\_2EBFT \ (ty\_2Elist\_2Elist \\ & \quad A\_27a) \ A\_27a) \ V0G) \ (c\_2Elist\_2ECONS \ A\_27a)) \ V2seen) \ V3fringe) \\ & \quad (c\_2Elist\_2ENIL \ A\_27a)))))))))) \end{aligned}$$