

thm_2Ebinary_ieee_2Edatatype_fp_op
(TMJ88x6tqrCiurRSfue2UMDQxk1ywfNxBMb)

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Let $ty_2Ebinary_ieee_2Efp_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efp_op\ A0\ A1) \quad (1)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (2)$$

Let $ty_2Ebinary_ieee_2Errounding : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Errounding \quad (3)$$

Let $c_2Ebinary_ieee_2EFP_MulSub : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow A_27t\ A_27w \in (((((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)}))^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)} \quad (4)$$

Let $c_2Ebinary_ieee_2EFP_MulAdd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow A_27t\ A_27w \in (((((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)}))^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)} \quad (5)$$

Let $c_2Ebinary_ieee_2EFP_Div : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Div\ A_27t\ A_27w \in (((((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)}))^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)} \quad (6)$$

Let $c_2Ebinary_ieee_2EFP_Mul : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Mul\ A_27t\ A_27w \in (((((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)}))^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Errounding)} \quad (7)$$

Let $c_2Ebinary_ieee_2EFP_Sub : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Sub\ A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)} \quad (8)$$

Let $c_2Ebinary_ieee_2EFP_Add : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Add\ A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)} \quad (9)$$

Let $c_2Ebinary_ieee_2EFP_Sqrt : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Sqrt\ A_27t\ A_27w \in (((ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)} \quad (10)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$.

Definition 3 We define $c_2Ebool_2EDATATYPE$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 4 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V0x \in 2^{A_27a}.V0x))\ (\lambda V1x \in 2^{A_27a}.V1x)))$.

Assume the following.

$$True \quad (11)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((p\ (ap\ (c_2Ebool_2EDATATYPE\ A_27a)\ V0x)) \Leftrightarrow True)) \quad (12)$$

Theorem 1

$$\forall V0fp_op \in ((((((2^{(ty_2Ebinary_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}$$