

# thm\_2Ebinary\_ieee\_2Eflags\_literal\_11 (TM-PVCmx1APoCxpJAK3AUCmEigFQyMnN9kTL)

October 26, 2020

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ebool\_2EARB\ A.27a \in A.27a \quad (1)$$

Let  $ty\_2Ebinary\_ieee\_2Eflags : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Eflags \quad (2)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (3)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (4)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (5)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (6)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (7)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (8)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding \in (2^{ty\_2Ebinary\_ieee\_2Eflags}) \quad (9)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding \in (2^{ty\_2Ebinary\_ieee\_2Eflags}) \quad (10)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Precision : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Precision \in (2^{ty\_2Ebinary\_ieee\_2Eflags}) \quad (11)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Overflow : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Overflow \in (2^{ty\_2Ebinary\_ieee\_2Eflags}) \quad (12)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_InvalidOp : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_InvalidOp \in (2^{ty\_2Ebinary\_ieee\_2Eflags}) \quad (13)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_DivideByZero : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_DivideByZero \in (2^{ty\_2Ebinary\_ieee\_2Eflags}) \quad (14)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow P \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define  $c\_2Ecombin\_2E\_2K$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Assume the following.

$$\begin{aligned}
& ((\forall V0f0 \in (2^2).(\forall V1f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero (ap (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V0f0) V1f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V1f)))))) \wedge ((\forall V2f0 \in (2^2).(\forall V3f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V2f0) V3f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V3f)))))) \wedge ((\forall V4f0 \in (2^2).(\forall V5f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V4f0) V5f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V5f)))))) \wedge ((\forall V6f0 \in (2^2).(\forall V7f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_Before V6f0) V7f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V7f)))))) \wedge ((\forall V8f0 \in (2^2).(\forall V9f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_After V8f0) V9f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V9f)))))) \wedge ((\forall V10f0 \in (2^2).(\forall V11f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp (ap (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V10f0) V11f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V11f)))))) \wedge \\
& ((\forall V12f0 \in (2^2).(\forall V13f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V12f0) V13f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V13f)))))) \wedge \\
& ((\forall V14f0 \in (2^2).(\forall V15f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V14f0) V15f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V15f)))))) \wedge \\
& ((\forall V16f0 \in (2^2).(\forall V17f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_Before V16f0) V17f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V17f)))))) \wedge \\
& ((\forall V18f0 \in (2^2).(\forall V19f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_After V18f0) V19f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V19f)))))) \wedge \\
& ((\forall V20f0 \in (2^2).(\forall V21f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow (ap (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V20f0) V21f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V21f)))))) \wedge \\
& ((\forall V22f0 \in (2^2).(\forall V23f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow (ap (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V22f0) V23f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V23f)))))) \wedge \\
& ((\forall V24f0 \in (2^2).(\forall V25f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V24f0) V25f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V25f)))))) \wedge \\
& ((\forall V26f0 \in (2^2).(\forall V27f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_Before V26f0) V27f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V27f)))))) \wedge \\
& ((\forall V28f0 \in (2^2).(\forall V29f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_After V28f0) V29f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V29f)))))) \wedge \\
& ((\forall V30f0 \in (2^2).(\forall V31f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision (ap (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V30f0) V31f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V31f)))))) \wedge \\
& ((\forall V32f0 \in (2^2).(\forall V33f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision (ap (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V32f0) V33f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V33f)))))) \wedge \\
& ((\forall V34f0 \in (2^2).(\forall V35f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V34f0) V35f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V35f)))))) \wedge \\
& ((\forall V36f0 \in (2^2).(\forall V37f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_Before V36f0) V37f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V37f)))))) \wedge \\
& ((\forall V38f0 \in (2^2).(\forall V39f \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_After V38f0) V39f))) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V39f)))))) \wedge
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f1 \in ty\_2Ebinary\_ieee\_2Eflags. (\forall V1f2 \in ty\_2Ebinary\_ieee\_2Eflags. \\
& ((V0f1 = V1f2) \Leftrightarrow (((p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero \\
& V0f1)) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero V1f2))) \wedge \\
& (((p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp V0f1)) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp \\
& V1f2))) \wedge (((p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V0f1)) \Leftrightarrow \\
& (p (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow V1f2))) \wedge (((p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision \\
& V0f1)) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Precision V1f2))) \wedge \\
& (((p (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding \\
& V0f1)) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding \\
& V1f2))) \wedge ((p (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding \\
& V0f1)) \Leftrightarrow (p (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding \\
& V1f2))))))))))))) \\
& \tag{16}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\
& \forall V0x \in A\_27a. (\forall V1y \in A\_27b. ((ap (ap (c\_2Ecombin\_2EK \\
& A\_27a A\_27b) V0x) V1y) = V0x))) \\
& \tag{17}
\end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0b41 \in 2. (\forall V1b31 \in 2. (\forall V2b21 \in 2. (\forall V3b11 \in \\
& 2. (\forall V4b01 \in 2. (\forall V5b1 \in 2. (\forall V6b42 \in 2. (\forall V7b32 \in \\
& 2. (\forall V8b22 \in 2. (\forall V9b12 \in 2. (\forall V10b02 \in 2. \\
& (\forall V11b2 \in 2. (((ap (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd \\
& (ap (c\_2Ecombin\_2EK 2 2) V0b41)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd \\
& (ap (c\_2Ecombin\_2EK 2 2) V1b31)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd \\
& (ap (c\_2Ecombin\_2EK 2 2) V2b21)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd \\
& (ap (c\_2Ecombin\_2EK 2 2) V3b11)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_ \\
& (ap (c\_2Ecombin\_2EK 2 2) V4b01)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_f \\
& (ap (c\_2Ecombin\_2EK 2 2) V5b1)) (c\_2Ebool\_2EARB ty\_2Ebinary\_ieee\_2Eflags))))))))) = \\
& (ap (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd (ap (c\_2Ecombin\_2EK \\
& 2 2) V6b42)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd \\
& (ap (c\_2Ecombin\_2EK 2 2) V7b32)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd \\
& (ap (c\_2Ecombin\_2EK 2 2) V8b22)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd \\
& (ap (c\_2Ecombin\_2EK 2 2) V9b12)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_ \\
& (ap (c\_2Ecombin\_2EK 2 2) V10b02)) (ap (ap c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_ \\
& (ap (c\_2Ecombin\_2EK 2 2) V11b2)) (c\_2Ebool\_2EARB ty\_2Ebinary\_ieee\_2Eflags))))))))) \Leftrightarrow \\
& (((p V0b41) \Leftrightarrow (p V6b42)) \wedge (((p V1b31) \Leftrightarrow (p V7b32)) \wedge (((p V2b21) \Leftrightarrow (p \\
& V8b22)) \wedge (((p V3b11) \Leftrightarrow (p V9b12)) \wedge (((p V4b01) \Leftrightarrow (p V10b02)) \wedge ((p V5b1) \Leftrightarrow \\
& (p V11b2))))))))))))) \\
& \tag{18}
\end{aligned}$$