thm_2Ebinary__ieee_2Efloat__accessors (TMb4nXnRYAYdPSX1RibXeQbotM9DeXqGQWg)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{2}$$

Let $ty\_2Ebinary\_\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A0\ A1) \tag{3}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{4}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{5}$$

Let $c\_2Ebinary\_\_ieee\_2Erecordtype\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Erecord$$
$$A\_27t\ A\_27w \in ((((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)})^{(ty\_2Efcp\_2Ecart\ 2\ A\_} \tag{6}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand$$
$$A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{7}$$

Assume the following.

$$\begin{array}{c}\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow (\\ \forall V0c \in (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V1c0 \in \\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w).(\forall V2c1 \in (ty\_2Efcp\_2Ecart\ 2 \\ A\_27t).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w)\ (ap \\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Erecordtype\_2Efloat\ A\_27t\ A\_27w) \\ V0c)\ V1c0)\ V2c1)) = V0c))))\end{array} \tag{8}$$

Assume the following.

$$\begin{array}{c}\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow (\\ \forall V0c \in (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V1c0 \in \\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w).(\forall V2c1 \in (ty\_2Efcp\_2Ecart\ 2 \\ A\_27t).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w) \\ (ap\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Erecordtype\_2Efloat\ A\_27t\ A\_27w) \\ V0c)\ V1c0)\ V2c1)) = V1c0))))\end{array} \tag{9}$$

Assume the following.

$$\begin{array}{c}\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow (\\ \forall V0c \in (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V1c0 \in \\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w).(\forall V2c1 \in (ty\_2Efcp\_2Ecart\ 2 \\ A\_27t).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w) \\ (ap\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Erecordtype\_2Efloat\ A\_27t\ A\_27w) \\ V0c)\ V1c0)\ V2c1)) = V2c1))))\end{array} \tag{10}$$

Theorem 1

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ($$
$$(\forall V0c \in (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V1c0 \in$$
$$(ty\_2Efcp\_2Ecart\ 2\ A\_27w).(\forall V2c1 \in (ty\_2Efcp\_2Ecart\ 2$$
$$A\_27t).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w)\ (ap$$
$$(ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Erecordtype\_2Efloat\ A\_27t\ A\_27w)$$
$$V0c)\ V1c0)\ V2c1)) = V0c))))\wedge((\forall V3c \in (ty\_2Efcp\_2Ecart\ 2$$
$$ty\_2Eone\_2Eone).(\forall V4c0 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27w).($$
$$\forall V5c1 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27t).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent$$
$$A\_27t\ A\_27w)\ (ap\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Erecordtype\_2Efloat$$
$$A\_27t\ A\_27w)\ V3c)\ V4c0)\ V5c1)) = V4c0))))\wedge(\forall V6c \in (ty\_2Efcp\_2Ecart$$
$$2\ ty\_2Eone\_2Eone).(\forall V7c0 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27w).$$
$$(\forall V8c1 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27t).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand$$
$$A\_27t\ A\_27w)\ (ap\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Erecordtype\_2Efloat$$
$$A\_27t\ A\_27w)\ V6c)\ V7c0)\ V8c1)) = V8c1))))))$$