

thm\_2Ebinary\_ieee\_2Efloat\_\_add\_\_finite  
(TMRhvTJCozXM4oayfhdFN2cJ213wxJxhyAm)

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Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (1)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (2)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (3)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

Let  $c\_2Ebinary\_ieee\_2EFloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EFloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (5)$$

Let  $ty\_2Ebinary\_ieee\_2Eflags : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Eflags \quad (6)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \quad (7)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A. \lambda x \in A. \lambda y \in A. inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A\_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x))$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (8)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (9)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (10)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (11)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (12)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (13)$$

**Definition 6** We define  $c\_2Ebinary\_ieee\_2Eclear\_flags$  to be  $(ap (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd))$

**Definition 7** We define  $c\_2Ebinary\_ieee\_2Einvalidop\_flags$  to be  $(ap (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd))$

Let  $ty\_2Ebinary\_ieee\_2Efp\_op : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efp\_op A0 A1) \quad (14)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efloat A0 A1) \quad (15)$$

Let  $ty\_2Ebinary\_ieee\_2Erunding : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Erunding \quad (16)$$

Let  $c\_2Ebinary\_ieee\_2EFP\_Add : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.\ nonempty\ A\_27t \Rightarrow \forall A\_27w.\ nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2EFP\_Add\ A\_27t\ A\_27w \in (((ty\_2Ebinary\_ieee\_2Efp\_op\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)} \quad (17)$$

Let  $ty\_2EfcP\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.\ nonempty\ A0 \Rightarrow \forall A1.\ nonempty\ A1 \Rightarrow nonempty\ (ty\_2EfcP\_2Ecart\ A0\ A1) \quad (18)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.\ nonempty\ A\_27t \Rightarrow \forall A\_27w.\ nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)} \quad (19)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (20)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (21)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (22)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (23)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (24)$$

**Definition 10** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ c\_2Enum\_2EABS\_num\ m)$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (25)$$

**Definition 11** We define `c_2Earithmetic_2EBIT1` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

**Definition 12** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (26)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (27)$$

Let  $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efcp\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (28)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (29)$$

Let  $ty\_2Efcp\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efcp\_2Efinite\_image A0) \quad (30)$$

**Definition 13** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 14** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E_7E$

**Definition 15** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E_21 2) (\lambda V2t \in 2$

**Definition 16** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x$  of type  $\iota \Rightarrow \iota$ ).

**Definition 17** We define `c_2Ebool_2E_3F` to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E_40$

**Definition 18** We define `c_2Eprim\_rec_2E_3C` to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 19** We define `c_2Ebool_2E_3F_21` to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E_2F_5C$

**Definition 20** We define `c_2Efcp_2Efinite\_index` to be  $\lambda A\_27a : \iota.(ap (c\_2Emin\_2E_40 (A\_27a^{ty\_2Enum\_2Enum}$

Let  $c\_2Efcp\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcp\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_image A\_27b)})^{(ty\_2Efcp\_2Ecart A\_27a A\_27b)}) \quad (31)$$

**Definition 21** We define `c_2Efcp_2Efcp\_index` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart A\_27a$

**Definition 22** We define  $c\_Ewords\_Eword\_msb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap$

**Definition 23** We define  $c\_Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ c\_Earithmetic$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (32)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (33)$$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (34)$$

**Definition 24** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

**Definition 25** We define  $c\_Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_Eboo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum}})^{ty\_2Enum\_2Enum}) \quad (35)$$

**Definition 26** We define  $c\_Ewords\_2Ew2n$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (36)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (37)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (38)$$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E.40\ (t$

Let  $c\_2Erealax\_2Etreal\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (39)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (40)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (41)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 29** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS)$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (42)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 31** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (43)$$

**Definition 32** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX\ A\_27a \in (ty\_2Eenum\_2Eenum^{(ty\_2Ebool\_2Eitself\ A\_27a)})(ty\_2Eenum\_2Eenum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (44)$$

Let  $c\_2Ebinary\_2Eieee\_2Efloat\_2Eexponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_2Eieee\_2Efloat\_2Eexponent\ A\_27t\ A\_27w \in ((ty\_2Efcpt\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_2Eieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (45)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (46)$$

Let  $c\_2Ebinary\_2Eieee\_2Efloat\_2Esign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_2Eieee\_2Efloat\_2Esign\ A\_27t\ A\_27w \in ((ty\_2Efcpt\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_2Eieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (47)$$

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (48)$$

**Definition 33** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_mul)$

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (49)$$

**Definition 34** We define  $c\_Ebit\_EDIV\_EXP$  to be  $\lambda V0x \in ty\_Enum\_Enum. \lambda V1n \in ty\_Enum\_Enum$

Let  $c\_Earithmetic\_EMOD : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EMOD \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \quad (50)$$

**Definition 35** We define  $c\_Ebit\_EMOD\_EXP$  to be  $\lambda V0x \in ty\_Enum\_Enum. \lambda V1n \in ty\_Enum\_Enum$

**Definition 36** We define  $c\_Ebit\_EBITS$  to be  $\lambda V0h \in ty\_Enum\_Enum. \lambda V1l \in ty\_Enum\_Enum. \lambda V$

**Definition 37** We define  $c\_Ebit\_EBIT$  to be  $\lambda V0b \in ty\_Enum\_Enum. \lambda V1n \in ty\_Enum\_Enum. (ap$

**Definition 38** We define  $c\_Efc\_EFCP$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0g \in (A\_27a^{ty\_Enum\_Enum}). (ap$

**Definition 39** We define  $c\_Ewords\_En2w$  to be  $\lambda A\_27a : \iota. \lambda V0n \in ty\_Enum\_Enum. (ap (c\_Efc\_EFCP$

**Definition 40** We define  $c\_Ebinary\_ieee\_Efloat\_to\_real$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_Ebinary\_$

Let  $c\_Ewords\_EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_Ewords\_EUINT\_MAX A\_27a \in ( \quad (51)$$

$$ty\_Enum\_Enum^{(ty\_Ebool\_Eitself A\_27a)})$$

**Definition 41** We define  $c\_Ewords\_Eword\_T$  to be  $\lambda A\_27a : \iota. (ap (c\_Ewords\_En2w A\_27a) (ap (c\_Ew$

**Definition 42** We define  $c\_Ebinary\_ieee\_Efloat\_value$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_Ebinary\_$

Let  $c\_Ebinary\_ieee\_Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty A\_27a \Rightarrow c\_Ebinary\_ieee\_Efloat\_value\_CASE$$

$$A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_Erealax\_Ereal})})^{ty\_Ebinary\_ieee\_Efloat\_value} \quad (52)$$

**Definition 43** We define  $c\_Ebinary\_ieee\_Efloat\_is\_nan$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_Ebinary\_$

**Definition 44** We define  $c\_Ebinary\_ieee\_Efloat\_is\_signalling$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_Ebinary\_$

**Definition 45** We define  $c\_Ebool\_ELET$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. (\lambda V0f \in (A\_27b^{A\_27a}). (\lambda V1x \in A\_27$

**Definition 46** We define  $c\_Ebinary\_ieee\_Efloat\_some\_qnan$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0fp\_op \in (ty$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Elist\_2Elist\ A0) \quad (53)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (54)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (55)$$

Let  $c\_2Elist\_2EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2EEXISTS\ A\_27a \in ((2^{(ty\_2Elist\_2Elist\ A\_27a)})^{(2^{A\_27a})}) \quad (56)$$

**Definition 47** We define  $c\_2Ebinary\_ieee\_2Echeck\_for\_signalling$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0l \in (ty\_2Elist\_2Elist\ A\_27a)$

Let  $c\_2Ebinary\_ieee\_2EroundTowardNegative : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EroundTowardNegative \in ty\_2Ebinary\_ieee\_2Erounding \quad (57)$$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (58)$$

**Definition 48** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 49** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 50** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ (ap\ (c\_2Ebool\_2ECONJ\ x))))$

Let  $c\_2Ebinary\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Elargest\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (59)$$

**Definition 51** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_finite$  to be  $\lambda A\_27t : \iota. \lambda A\_27w : \iota. \lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27t\ A\_27w)$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (60)$$



**Definition 52** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (61)$$

**Definition 53** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 54** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

**Definition 55** We define  $c\_2Ebinary\_ieee\_2Eis\_closest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebinary\_ieee\_2Efloat\_top\ A\_27a\ A\_27b)})$

**Definition 56** We define  $c\_2Ebinary\_ieee\_2Eclosest\_such$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebinary\_ieee\_2Efloat\_top\ A\_27a\ A\_27b)})$

**Definition 57** We define  $c\_2Ebinary\_ieee\_2Eclosest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap (c\_2Ebinary\_ieee\_2Efloat\_top\ A\_27a\ A\_27b))$

Let  $c\_2Ebinary\_ieee\_2Efloat\_top : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_top \\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (62)$$

**Definition 58** We define  $c\_2Ereal\_2Ereal\_gt$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ebinary\_ieee\_2Efloat\_bottom : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_bottom \\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (63)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity \\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (64)$$

**Definition 59** We define  $c\_2Ereal\_2Ereal\_ge$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity \\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (65)$$

Let  $c\_2Ebinary\_ieee\_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Ethreshold \\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (66)$$

**Definition 60** We define  $c\_Ewords\_Eword\_lsb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).(ap$   
 Let  $c\_2Ebinary\_ieee\_2Erounding2num : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_ieee\_2Erounding}) \quad (67)$$

**Definition 61** We define  $c\_2Ebinary\_ieee\_2Erounding\_CASE$  to be  $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_ieee\_2E$

**Definition 62** We define  $c\_2Ebinary\_ieee\_2Eround$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_ie$

Let  $c\_2Ebinary\_ieee\_2Efloat\_plus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_plus\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)})) \quad (68)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_minus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_minus\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)})) \quad (69)$$

**Definition 63** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_zero$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

**Definition 64** We define  $c\_2Ebinary\_ieee\_2Efloat\_round$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebin$

Let  $c\_2Ewords\_2EINT\_MIN : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_MIN\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (70)$$

Let  $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (71)$$

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (72)$$

**Definition 65** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).$

**Definition 66** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 67** We define  $c\_2Ewords\_2Enzcv$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Efc\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (73)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \end{aligned} \quad (74)$$

**Definition 68** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a})$

**Definition 69** We define  $c\_2Ewords\_2Eword\_ls$  to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1b$

**Definition 70** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_infinite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

**Definition 71** We define  $c\_2Ebinary\_ieee\_2Efloat\_round\_with\_flags$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode$

**Definition 72** We define  $c\_2Epair\_2Epair\_CASE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair$

**Definition 73** We define  $c\_2Ebinary\_ieee\_2Efloat\_add$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebin$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in ty\_2Erealax\_2Ereal. \\ (\forall V1f \in (A\_27a)^{ty\_2Erealax\_2Ereal}).(\forall V2v \in A\_27a. \\ (\forall V3v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\ A\_27a)\ (ap\ c\_2Ebinary\_ieee\_2Efloat\ V0a))\ V1f)\ V2v)\ V3v1) = (ap \\ V1f\ V0a)))))) \wedge ((\forall V4f \in (A\_27a)^{ty\_2Erealax\_2Ereal}).(\forall V5v \in \\ A\_27a.(\forall V6v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\ A\_27a)\ c\_2Ebinary\_ieee\_2EInfinity)\ V4f)\ V5v)\ V6v1) = V5v)))) \wedge \\ (\forall V7f \in (A\_27a)^{ty\_2Erealax\_2Ereal}).(\forall V8v \in A\_27a. \\ (\forall V9v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\ A\_27a)\ c\_2Ebinary\_ieee\_2ENaN)\ V7f)\ V8v)\ V9v1) = V9v1)))))) \end{aligned} \quad (75)$$

Assume the following.

$$True \quad (76)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (77)$$

Assume the following.

$$\begin{aligned} (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (( \\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (78)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (79)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (80)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (81)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x\_27 \in 2. (\forall V2y \in 2. (\forall V3y\_27 \in 2. (((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))))) \quad (82)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & V5y\_27)))))))))) \quad (83) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c. \\ & nonempty\ A\_27c \Rightarrow (\forall V0x \in A\_27b. (\forall V1y \in A\_27c. (\forall V2f \in \\ & ((A\_27a^{A\_27c})^{A\_27b}). ((ap\ (ap\ (c\_2Epair\_2Epair\_CASE\ A\_27a\ A\_27b \\ & A\_27c)\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ A\_27b\ A\_27c)\ V0x)\ V1y))\ V2f) = (ap \\ & (ap\ V2f\ V0x)\ V1y)))))) \quad (84) \end{aligned}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\
& \quad \forall V0mode \in ty\_2Ebinary\_ieee\_2Erounding. (\forall V1x \in \\
& \quad (ty\_2Ebinary\_ieee\_2Efloat\ A\_27a\ A\_27b). (\forall V2y \in (ty\_2Ebinary\_ieee\_2Efloat \\
& \quad A\_27a\ A\_27b). (\forall V3r1 \in ty\_2Erealx\_2Ereal. (\forall V4r2 \in \\
& \quad ty\_2Erealx\_2Ereal. (((ap\ (c\_2Ebinary\_ieee\_2Efloat\_value \\
& \quad A\_27a\ A\_27b)\ V1x) = (ap\ c\_2Ebinary\_ieee\_2Efloat\ V3r1)) \wedge ((ap\ ( \\
& \quad c\_2Ebinary\_ieee\_2Efloat\_value\ A\_27a\ A\_27b)\ V2y) = (ap\ c\_2Ebinary\_ieee\_2Efloat \\
& \quad V4r2))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_add\ A\_27a\ A\_27b) \\
& \quad V0mode)\ V1x)\ V2y) = (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_round\_with\_flags \\
& \quad A\_27a\ A\_27b)\ V0mode)\ (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ 2)\ (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C \\
& \quad (ap\ (ap\ (c\_2Emin\_2E\_3D\ ty\_2Erealx\_2Ereal)\ V3r1)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)))\ (ap\ (ap\ c\_2Ebool\_2E\_2F\_5C\ (ap\ (ap\ (c\_2Emin\_2E\_3D \\
& \quad ty\_2Erealx\_2Ereal)\ V4r2)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))) \\
& \quad (ap\ (ap\ (c\_2Emin\_2E\_3D\ (ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone))\ ( \\
& \quad ap\ (c\_2Ebinary\_ieee\_2Efloat\_Sign\ A\_27a\ A\_27b)\ V1x))\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_Sign \\
& \quad A\_27a\ A\_27b)\ V2y))))\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (ty\_2Efc\_2Ecart \\
& \quad 2\ ty\_2Eone\_2Eone))\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_Sign\ A\_27a \\
& \quad A\_27b)\ V1x))\ (ap\ (c\_2Ewords\_2En2w\ ty\_2Eone\_2Eone)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))\ (ap\ (ap \\
& \quad (c\_2Emin\_2E\_3D\ ty\_2Ebinary\_ieee\_2Erounding)\ V0mode)\ c\_2Ebinary\_ieee\_2EroundTowardNegative) \\
& \quad (ap\ (ap\ c\_2Erealx\_2Ereal\_add\ V3r1\ V4r2)))))))))
\end{aligned}$$