thm_2Ebinary__ieee_2Efloat__add__finite__minus__infinity (TMbFL4ephRR9xsPRmNpcWcUm8fKdsisbW8F)

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Let $ty_2Ebinary__ieee_2Eflags : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary__ieee_2Eflags \tag{1}$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \tag{2}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\mathsf{c_2Emin_2E_3D}\ (2^{A_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Ecombin_2EK *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Let $c_2Ebinary__ieee_2Eflags__Underflow__AfterRounding__fupd : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2Eflags__Underflow__AfterRounding__fupd \in ((ty_2Ebinary__ieee_2Eflags^{ty_2Ebinary__ieee_2Eflags})^{(2^2)}) \tag{3}$$

Let $c_2Ebinary__ieee_2Eflags__Underflow__BeforeRounding__fupd : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2Eflags__Underflow__BeforeRounding__fupd \in ((ty_2Ebinary__ieee_2Eflags^{ty_2Ebinary__ieee_2Eflags})^{(2^2)}) \tag{4}$$

Let $c_2Ebinary__ieee_2Eflags__Precision__fupd : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2Eflags__Precision__fupd \in ((ty_2Ebinary__ieee_2Eflags^{ty_2Ebinary__ieee_2Eflags})^{(2^2)}) \tag{5}$$

Let $c_2Ebinary__ieee_2Eflags__Overflow__fupd : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2Eflags__Overflow__fupd \in ((ty_2Ebinary__ieee_2Eflags^{ty_2Ebinary__ieee_2Eflags})^{(2^2)}) \tag{6}$$

Let $c_2Ebinary__ieee_2Eflags__InvalidOp__fupd : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2Eflags__InvalidOp__fupd \in ((ty_2Ebinary__ieee_2Eflags^{ty_2Ebinary__ieee_2Eflags})^{(2^2)}) \tag{7}$$

Let $c_2Ebinary__ieee_2Eflags__DivideByZero__fupd : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2Eflags__DivideByZero__fupd \in ((ty_2Ebinary__ieee_2Eflags^{ty_2Ebinary__ieee_2Eflags} \tag{8}$$

Definition 6 *We define* c_2Ebinary__ieee_2Eclear__flags *to be* $(ap\ (ap\ c_2Ebinary__ieee_2Eflags__DivideBy$

Definition 7 *We define* c_2Ebinary__ieee_2Einvalidop__flags *to be* $(ap\ (ap\ c_2Ebinary__ieee_2Eflags__Inval$

Let $ty_2Ebinary__ieee_2Efp__op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary__ieee_2Efp__op\ A0\ A1) \tag{9}$$

Let $ty_2Ebinary__ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary__ieee_2Efloat\ A0\ A1) \tag{10}$$

Let $ty_2Ebinary__ieee_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary__ieee_2Erounding \tag{11}$$

Let $c_2Ebinary__ieee_2EFP__Add : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary$$
$$A_27t\ A_27w \in ((((ty_2Ebinary__ieee_2Efp__op\ A_27t\ A_27w)^{(ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2El} \tag{12}$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \tag{13}$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \tag{14}$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \tag{15}$$

Let $ty_2Efcp_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efcp_2Ecart\ A0\ A1) \tag{16}$$

Let $c_2Ebinary__ieee_2Efloat__Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__Significand\ A_27t\ A_27w \in ((ty_2Efcp_2Ecart\ 2\ A_27t)^{(ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)}) \tag{17}$$

Let $c_2Enum_2EZERO__REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO__REP \in omega \tag{18}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{19}$$

Let $c_2Enum_2EABS__num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS__num \in (ty_2Enum_2Enum^{omega}) \tag{20}$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c_2Enum_2EABS__num\ c_2Enum_2EZERO__REP)$.

Definition 9 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c_2Enum_2EREP__num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP__num \in (omega^{ty_2Enum_2Enum}) \tag{21}$$

Let $c_2Enum_2ESUC__REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC__REP \in (omega^{omega}) \tag{22}$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS__num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{23}$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 12 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \tag{24}$$

Let $c_2Ebool_2Ethe__value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe__value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \tag{25}$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcp_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \tag{26}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{27}$$

Let $ty_2Efcp_2Efinite__image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcp_2Efinite__image\ A0) \tag{28}$$

Definition 13 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 14 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 15 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 16 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 17 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 18 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enu$

Definition 19 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 20 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A_27a : \iota.(ap\ ($c_2Emin_2E_40 $(A_27a^{ty_2Enum_2Enu}$

Let $c_2Efcp_2Edest__cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest__cart\ A_27a\ A_27b \in ((A_27a^{(ty_2Efcp_2Efinite__image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \tag{29}$$

Definition 21 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart\ A_27$

Definition 22 *We define* c_2Ewords_2Eword__msb *to be* $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(a$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{30}$$

Definition 23 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Ereal_2Ereal__of__num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal__of__num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \tag{31}$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \tag{32}$$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{33}$$

Definition 24 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 25 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ ($c_2Eboo

Let $c_2Esum__num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum__num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \tag{34}$$

Definition 26 *We define* c_2Ewords_2Ew2n *to be* $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(ap\ (ap\ c$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{35}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{36}$$

Let $c_2Erealax_2Ereal__REP__CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal__REP__CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax} \tag{37}$$

Definition 27 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ ($c_2Emin_2E_40 $(t$

Let $c_2Erealax_2Etreal__inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \tag{38}$$

Let $c_2Erealax_2Etreal__eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Eh} \tag{39}$$

Let $c_2Erealax_2Ereal__ABS__CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal__ABS__CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Eh} \tag{40}$$

Definition 28 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty$

Definition 29 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c_2Erealax_2Etreal__mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Eh} \tag{41}$$

Definition 30 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Ereala$

Definition 31 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Let $c_2Erealax_2Etreal__add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Eh} \tag{42}$$

Definition 32 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Ereala$

Let $c_2Ewords_2EINT__MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT__MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_2} \tag{43}$$

Let $c_2Ebinary__ieee_2Efloat__Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__Exponent\ A_27t\ A_27w \in ((ty_2Efcp_2Ecart\ 2\ A_27w)^{(ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)}) \tag{44}$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \tag{45}$$

Let $c_2Ebinary__ieee_2Efloat__Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__Sign\ A_27t\ A_27w \in ((ty_2Efcp_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)}) \tag{46}$$

Let $c_2Erealax_2Etreal__neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \tag{47}$$

Definition 33 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{48}$$

Definition 34 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2En$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{49}$$

Definition 35 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2E$

Definition 36 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 37 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 38 *We define* c_2Efcp_2EFCP *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 39 *We define* c_2Ewords_2En2w *to be* $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap$ (c_2Efcp_2EFC

Definition 40 *We define* c_2Ebinary__ieee_2Efloat__to__real *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Let $ty_2Ebinary__ieee_2Efloat__value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary__ieee_2Efloat__value \tag{50}$$

Let $c_2Ebinary__ieee_2EFloat : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2EFloat \in (ty_2Ebinary__ieee_2Efloat__value^{ty_2Erealax_2Ereal}) \tag{51}$$

Let $c_2Ebinary__ieee_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2ENaN \in ty_2Ebinary__ieee_2Efloat__value \tag{52}$$

Let $c_2Ebinary__ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2EInfinity \in ty_2Ebinary__ieee_2Efloat__value \tag{53}$$

Let $c_2Ewords_2EUINT__MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT__MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \tag{54}$$

Definition 41 *We define* c_2Ewords_2Eword__T *to be* $\lambda A_27a : \iota.(ap$ (c_2Ewords_2En2w A_27a) $(ap$ $(c_2Ew$

Definition 42 *We define* c_2Ebinary__ieee_2Efloat__value *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_$

Let $c_2Ebinary__ieee_2Efloat__value__CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary__ieee_2Efloat__value__CASE\ A_27a \in ((((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary__ieee_2Efloat__value}) \tag{55}$$

Definition 43 *We define* c_2Ebinary__ieee_2Efloat__is__nan *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Definition 44 *We define* c_2Ebinary__ieee_2Efloat__is__signalling *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2E$

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEXISTS\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \tag{56}$$

Definition 45 *We define* c_2Ebinary__ieee_2Echeck__for__signalling *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0l \in (ty_2E$

Let $c_2Ebinary__ieee_2EroundTowardNegative : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2EroundTowardNegative \in ty_2Ebinary__ieee_2Erounding \tag{57}$$

Let $c_2Erealax_2Etreal__lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal__lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Eh} \tag{58}$$

Definition 46 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_$

Definition 47 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Er$

Definition 48 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ ($c_2Ebool_2ECON

Let $c_2Ebinary__ieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Elargest\ A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \tag{59}$$

Definition 49 *We define* c_2Ebinary__ieee_2Efloat__is__finite *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let $c_2Epair_2EABS__prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS__prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{((2^{A_27b})^{A_27a})}) \tag{60}$$

Definition 50 *We define* c_2Epair_2E_2C *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_2Epred__set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred__set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b})}) \tag{61}$$

Definition 51 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2E$

Definition 52 *We define* c_2Ebool_2EIN *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x))$

Definition 53 *We define* c_2Ebinary__ieee_2Eis__closest *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_2Ebinary__ie}$

Definition 54 *We define* c_2Ebinary__ieee_2Eclosest__such *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Ebinary}$

Definition 55 *We define* c_2Ebinary__ieee_2Eclosest *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap$ (c_2Ebinary__ieee_2Eclos

Let $c_2Ebinary__ieee_2Efloat__top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__top\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{62}$$

Definition 56 *We define* c_2Ereal_2Ereal__gt *to be* $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ere$

Let $c_2Ebinary__ieee_2Efloat__bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__bottom\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{63}$$

Definition 57 *We define* c_2Ebool_2ELET *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27$

Let $c_2Ebinary__ieee_2Efloat__minus__infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__minus__infinity\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{64}$$

Definition 58 *We define* c_2Ereal_2Ereal__ge *to be* $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ere$

Let $c_2Ebinary__ieee_2Efloat__plus__infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__plus__infinity\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{65}$$

Let $c_2Ebinary__ieee_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Ethreshold\ A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \tag{66}$$

Definition 59 *We define* c_2Ewords_2Eword__lsb *to be* $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).(ap$

Let $c_2Ebinary__ieee_2Erounding2num : \iota$ be given. Assume the following.

$$c_2Ebinary__ieee_2Erounding2num \in (ty_2Enum_2Enum^{ty_2Ebinary__ieee_2Erounding}) \tag{67}$$

Definition 60 *We define* c_2Ebinary__ieee_2Erounding__CASE *to be* $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinary__ieee_2$

Definition 61 *We define* c_2Ebinary__ieee_2Eround *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary__ie$

Let $c_2Ebinary__ieee_2Efloat__plus__zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__plus__zero\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{68}$$

Let $c_2Ebinary__ieee_2Efloat__minus__zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__minus__zero\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{69}$$

Definition 62 *We define* c_2Ebinary__ieee_2Efloat__is__zero *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Definition 63 *We define* c_2Ebinary__ieee_2Efloat__round *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebin$

Let $c_2Ewords_2EINT__MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT__MIN\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27} \tag{70}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{71}$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \tag{72}$$

Definition 64 *We define* c_2Ewords_2Eword__2comp *to be* $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a).$

Definition 65 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 66 *We define* c_2Ewords_2Enzcv *to be* $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efcp_2Ecart\ 2\ A_27a).\lambda V1b \in ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{73}$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \tag{74}$$

Definition 67 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27}$

Definition 68 *We define* c_2Ewords_2Eword__ls *to be* $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efcp_2Ecart\ 2\ A_27a).\lambda V1b$

Definition 69 *We define* c_2Ebinary__ieee_2Efloat__is__infinite *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebin$

Definition 70 *We define* c_2Ebinary__ieee_2Efloat__round__with__flags *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode$

Definition 71 *We define* c_2Ebinary__ieee_2Efloat__some__qnan *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0fp__op \in (ty$

Definition 72 *We define* c_2Epair_2Epair__CASE *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair_$

Definition 73 *We define* c_2Ebinary__ieee_2Efloat__add *to be* $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinar$

Let $c_2Ebinary__ieee_2Efloat__minus__min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__minus__min\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{75}$$

Let $c_2Ebinary__ieee_2Efloat__plus__min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary__ieee_2Efloat__plus__min\ A_27t\ A_27w \in ((ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \tag{76}$$

Assume the following.

$$\begin{gathered}\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in ty_2Erealax_2Ereal.\\ (\forall V1f \in (A_27a^{ty_2Erealax_2Ereal}).(\forall V2v \in A_27a.\\ (\forall V3v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary__ieee_2Efloat__value__CASE\\ A_27a)\ (ap\ c_2Ebinary__ieee_2EFloat\ V0a))\ V1f)\ V2v)\ V3v1) = (ap\\ V1f\ V0a))))))\wedge((\forall V4f \in (A_27a^{ty_2Erealax_2Ereal}).(\forall V5v \in\\ A_27a.(\forall V6v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary__ieee_2Efloat__value__CASE\\ A_27a)\ c_2Ebinary__ieee_2EInfinity)\ V4f)\ V5v)\ V6v1) = V5v))))\wedge\\ (\forall V7f \in (A_27a^{ty_2Erealax_2Ereal}).(\forall V8v \in A_27a.\\ (\forall V9v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary__ieee_2Efloat__value__CASE\\ A_27a)\ c_2Ebinary__ieee_2ENaN)\ V7f)\ V8v)\ V9v1) = V9v1))))))\end{gathered} \tag{77}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27t.$$
$$nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}$$
$$A_27t\ A_27w)\ (ap\ (c_2Ebinary__ieee_2Efloat__plus__infinity$$
$$A_27t\ A_27w)\ (c_2Ebool_2Ethe__value\ (ty_2Epair_2Eprod\ A_27t$$
$$A_27w)))) = c_2Ebinary__ieee_2EInfinity) \wedge (((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}$$
$$A_27t\ A_27w)\ (ap\ (c_2Ebinary__ieee_2Efloat__minus__infinity$$
$$A_27t\ A_27w)\ (c_2Ebool_2Ethe__value\ (ty_2Epair_2Eprod\ A_27t$$
$$A_27w)))) = c_2Ebinary__ieee_2EInfinity) \wedge ((\forall V0fp__op \in$$
$$(ty_2Ebinary__ieee_2Efp__op\ A_27a\ A_27b).((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}$$
$$A_27a\ A_27b)\ (ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__some__qnan}\ A_27a$$
$$A_27b)\ V0fp__op)) = c_2Ebinary__ieee_2ENaN)) \wedge (((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}$$
$$A_27t\ A_27w)\ (ap\ (c_2Ebinary__ieee_2Efloat__plus__zero\ A_27t$$
$$A_27w)\ (c_2Ebool_2Ethe__value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))) =$$
$$(ap\ c_2Ebinary__ieee_2EFloat\ (ap\ c_2Ereal_2Ereal__of__num\ \mathsf{c_2Enum_2E0}))) \wedge$$
$$(((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}\ A_27t\ A_27w)\ (ap\ (c_2Ebinary__ieee_2Efloat__minus__zero$$
$$A_27t\ A_27w)\ (c_2Ebool_2Ethe__value\ (ty_2Epair_2Eprod\ A_27t$$
$$A_27w)))) = (ap\ c_2Ebinary__ieee_2EFloat\ (ap\ c_2Ereal_2Ereal__of__num$$
$$\mathsf{c_2Enum_2E0}))) \wedge (((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}\ A_27t$$
$$A_27w)\ (ap\ (c_2Ebinary__ieee_2Efloat__plus__min\ A_27t\ A_27w)$$
$$(c_2Ebool_2Ethe__value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))) = ($$
$$ap\ c_2Ebinary__ieee_2EFloat\ (ap\ (ap\ \mathsf{c_2Ereal_2E_2F}\ (ap\ c_2Ereal_2Ereal__of__num$$
$$(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ \mathsf{c_2Earithmetic_2EZERO}))))$$
$$(ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ereal_2Ereal__of__num\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}$$
$$(ap\ \mathsf{c_2Earithmetic_2EBIT2}\ \mathsf{c_2Earithmetic_2EZERO}))))\ (ap\ (ap$$
$$c_2Earithmetic_2E_2B\ (ap\ (c_2Ewords_2EINT__MAX\ A_27w)\ (c_2Ebool_2Ethe__value$$
$$A_27w)))\ (ap\ (c_2Efcp_2Edimindex\ A_27t)\ (c_2Ebool_2Ethe__value$$
$$A_27t))))))) \wedge ((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}\ A_27t\ A_27w)$$
$$(ap\ (c_2Ebinary__ieee_2Efloat__minus__min\ A_27t\ A_27w)\ (c_2Ebool_2Ethe__value$$
$$(ty_2Epair_2Eprod\ A_27t\ A_27w)))) = (ap\ c_2Ebinary__ieee_2EFloat$$
$$(ap\ (ap\ \mathsf{c_2Ereal_2E_2F}\ (ap\ \mathsf{c_2Erealax_2Ereal__neg}\ (ap\ c_2Ereal_2Ereal__of__num$$
$$(ap\ \mathsf{c_2Earithmetic_2ENUMERAL}\ (ap\ \mathsf{c_2Earithmetic_2EBIT2}\ \mathsf{c_2Earithmetic_2EZERO}))))$$
$$(ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ereal_2Ereal__of__num\ (ap\ \mathsf{c_2Earithmetic_2ENUMERAL}$$
$$(ap\ \mathsf{c_2Earithmetic_2EBIT2}\ \mathsf{c_2Earithmetic_2EZERO}))))\ (ap\ (ap$$
$$c_2Earithmetic_2E_2B\ (ap\ (c_2Ewords_2EINT__MAX\ A_27w)\ (c_2Ebool_2Ethe__value$$
$$A_27w)))\ (ap\ (c_2Efcp_2Edimindex\ A_27t)\ (c_2Ebool_2Ethe__value$$
$$A_27t))))))))))))) \tag{78}$$

Assume the following.

$$True \tag{79}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{80}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{81}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{82}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{83}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{84}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{85}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))))) \tag{86}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a.(\forall V5y_27 \in A_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c_2Ebool_2ECOND}\ A_27a)\ V1Q)\ V3x_27)\ V5y_27))))))))) \tag{87}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in A_27b.(((ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b))))))) \tag{88}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c.\ nonempty\ A_27c \Rightarrow (\forall V0x \in A_27b.(\forall V1y \in A_27c.(\forall V2f \in ((A_27a^{A_27c})^{A_27b}).((ap\ (ap\ (\mathsf{c_2Epair_2Epair__CASE}\ A_27a\ A_27b\ A_27c)\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27b\ A_27c)\ V0x)\ V1y))\ V2f) = (ap\ (ap\ V2f\ V0x)\ V1y))))) \tag{89}$$

Theorem 1

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\forall V0mode \in ty_2Ebinary__ieee_2Erounding.(\forall V1x \in (ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w).(\forall V2r \in ty_2Erealax_2Ereal.(((ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__value}\ A_27t\ A_27w)\ V1x) = (ap\ c_2Ebinary__ieee_2EFloat\ V2r)) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c_2Ebinary__ieee_2Efloat__add}\ A_27t\ A_27w)\ V0mode)\ V1x)\ (ap\ (c_2Ebinary__ieee_2Efloat__minus__infinity\ A_27t\ A_27w)\ (c_2Ebool_2Ethe__value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))) = (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ ty_2Ebinary__ieee_2Eflags\ (ty_2Ebinary__ieee_2Efloat\ A_27t\ A_27w))\ \mathsf{c_2Ebinary__ieee_2Eclear__flags})\ (ap\ (c_2Ebinary__ieee_2Efloat__minus__infinity\ A_27t\ A_27w)\ (c_2Ebool_2Ethe__value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))))))$$