

thm_2Ebinary_ieee_2Efloat__add__finite__plus__infinity
 (TM-
 NtdW466J11TfpHVXNCYqQZdfbwqyvoFbd)

October 26, 2020

Let $ty_2Ebinary_ieee_2Eflags : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Eflags \quad (1)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ebool_2EARB\ A.27a \in A.27a \quad (2)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 3 We define c_2Ebool_2E21 to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))))$

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x))$

Let $c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (3)$$

Let $c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (4)$$

Let $c_2Ebinary_ieee_2Eflags_Precision_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Precision_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (5)$$

Let $c_2Ebinary_ieee_2Eflags_Overflow_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Overflow_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (6)$$

Let $c_2Ebinary_ieee_2Eflags_InvalidOp_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_InvalidOp_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (7)$$

Let $c_2Ebinary_ieee_2Eflags_DivideByZero_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_DivideByZero_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (8)$$

Definition 6 We define $c_2Ebinary_ieee_2Eclear_flags$ to be $(ap (ap c_2Ebinary_ieee_2Eflags_DivideByZero_fupd))$

Definition 7 We define $c_2Ebinary_ieee_2Einvalidop_flags$ to be $(ap (ap c_2Ebinary_ieee_2Eflags_InvalidOp_fupd))$

Let $ty_2Ebinary_ieee_2Efp_op : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieee_2Efp_op A0 A1) \quad (9)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieee_2Efloat A0 A1) \quad (10)$$

Let $ty_2Ebinary_ieee_2Errounding : \iota$ be given. Assume the following.

$$nonempty ty_2Ebinary_ieee_2Errounding \quad (11)$$

Let $c_2Ebinary_ieee_2EFP_Add : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2EFP_Add A_27t A_27w \in (((ty_2Ebinary_ieee_2Efp_op A_27t A_27w)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)})^{(ty_2Errounding)}) \quad (12)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Elist_2Elist A0) \quad (13)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (14)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (15)$$

Let $ty_2EfcP_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2EfcP_2Ecart\ A0\ A1) \quad (16)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (17)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (18)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (19)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (20)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (21)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (22)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (23)$$

Definition 11 We define $c_Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic$

Definition 12 We define $c_Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Ebool_2Eitself A0) \quad (24)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ebool_2Ethe_value A_27a \in (ty_2Ebool_2Eitself A_27a) \quad (25)$$

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcf_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (26)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (27)$$

Let $ty_2Efcf_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Efcf_2Efinite_image A0) \quad (28)$$

Definition 13 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 14 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 15 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Definition 16 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge p x)$ of type $\iota \Rightarrow \iota$).

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

Definition 18 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 19 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

Definition 20 We define $c_2Efcf_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let $c_2Efcf_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcf_2Edest_cart A_27a A_27b \in ((A_27a^{(ty_2Efcf_2Efinite_image A_27b)})^{(ty_2Efcf_2Ecart A_27a A_27b)}) \quad (29)$$

Definition 21 We define $c_2Efcf_2Efcf_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcf_2Ecart A_27a$

Definition 22 We define $c_Ewords_Eword_msb$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A.27a).(ap$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (30)$$

Definition 23 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (31)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (32)$$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (33)$$

Definition 24 We define c_2Ebool_2ECOND to be $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

Definition 25 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebo$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (34)$$

Definition 26 We define $c_2Ewords_2Ew2n$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A.27a).(ap\ (ap\ c$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (35)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (36)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (37)$$

Definition 27 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E.40\ (t$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (38)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (39)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (40)$$

Definition 28 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 29 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (41)$$

Definition 30 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 31 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (42)$$

Definition 32 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum)^{(ty_2Ebool_2Eitself\ A_27a)} \quad (43)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (44)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (45)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (46)$$

Let $c_2Erealax_2Etreax_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (47)$$

Definition 33 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (48)$$

Definition 34 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (49)$$

Definition 35 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 36 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 37 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 38 We define c_2Efc_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 39 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efc_2EFCP$

Definition 40 We define $c_2Ebinary_ieee_2Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Let $ty_2Ebinary_ieee_2Efloat_value : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Efloat_value \quad (50)$$

Let $c_2Ebinary_ieee_2Efloat : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Efloat \in (ty_2Ebinary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (51)$$

Let $c_2Ebinary_ieee_2ENaN : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2ENaN \in ty_2Ebinary_ieee_2Efloat_value \quad (52)$$

Let $c_2Ebinary_ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2EInfinity \in ty_2Ebinary_ieee_2Efloat_value \quad (53)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (54)$$

Definition 41 We define $c_Ewords_Eword_T$ to be $\lambda A_27a : \iota.(ap (c_Ewords_Een2w A_27a) (ap (c_Ew$

Definition 42 We define $c_Ebinary_ieee_Efloat_value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_EBinary_$

Let $c_EBinary_ieee_Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow c_EBinary_ieee_Efloat_value_CASE \\ & A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_ERealax_EReal})})^{ty_EBinary_ieee_Efloat_value} \end{aligned} \quad (55)$$

Definition 43 We define $c_EBinary_ieee_Efloat_is_nan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_EBinary_$

Definition 44 We define $c_EBinary_ieee_Efloat_is_signalling$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_EBinary_$

Let $c_Elist_EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Elist_EEXISTS A_27a \in ((2^{(ty_Elist_Elist A_27a)})^{(2^{A_27a})}) \quad (56)$$

Definition 45 We define $c_EBinary_ieee_Echeck_for_signalling$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0l \in (ty_EBinary_$

Let $c_EBinary_ieee_ERoundTowardNegative : \iota$ be given. Assume the following.

$$c_EBinary_ieee_ERoundTowardNegative \in ty_EBinary_ieee_ERounding \quad (57)$$

Let $c_ERealax_EReal_lt : \iota$ be given. Assume the following.

$$c_ERealax_EReal_lt \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod ty_Ehreal_Ehreal)}) \quad (58)$$

Definition 46 We define $c_ERealax_EReal_lt$ to be $\lambda V0T1 \in ty_ERealax_EReal.\lambda V1T2 \in ty_ERealax_EReal$

Definition 47 We define $c_EReal_EReal_lte$ to be $\lambda V0x \in ty_ERealax_EReal.\lambda V1y \in ty_ERealax_EReal$

Definition 48 We define c_EReal_Eabs to be $\lambda V0x \in ty_ERealax_EReal.(ap (ap (ap (c_Ebool_ECONJ$

Let $c_EBinary_ieee_Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_EBinary_ieee_Elargest \\ & A_27t A_27w \in (ty_ERealax_EReal^{(ty_Ebool_Eitself (ty_Epair_Eprod A_27t A_27w))}) \end{aligned} \quad (59)$$

Definition 49 We define $c_EBinary_ieee_Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_EBinary_$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EABS_prod \\ & A_27a A_27b \in ((ty_Epair_Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (60)$$

Definition 50 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota)$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a A_27b \in ((2^{A_27a})^{((ty_2Epair_2Eprod A_27a 2)^{A_27b})}) \quad (61)$$

Definition 51 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 52 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap V1f V0x))$

Definition 53 We define $c_2Ebinary_ieee_2Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_2Ebinary_iee$

Definition 54 We define $c_2Ebinary_ieee_2Eclosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Ebinary_iee$

Definition 55 We define $c_2Ebinary_ieee_2Eclosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap (c_2Ebinary_ieee_2Eclose$

Let $c_2Ebinary_ieee_2Efloat_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_top \\ A_27t A_27w \in ((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebool_2Eitself (ty_2Epair_2Eprod A_27t A_27w))}) \quad (62)$$

Definition 56 We define $c_2Ereal_2Ereal_gt$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieee_2Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_bottom \\ A_27t A_27w \in ((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebool_2Eitself (ty_2Epair_2Eprod A_27t A_27w))}) \quad (63)$$

Definition 57 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27b$

Let $c_2Ebinary_ieee_2Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_infinity \\ A_27t A_27w \in ((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebool_2Eitself (ty_2Epair_2Eprod A_27t A_27w))}) \quad (64)$$

Definition 58 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieee_2Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_infinity \\ A_27t A_27w \in ((ty_2Ebinary_ieee_2Efloat A_27t A_27w)^{(ty_2Ebool_2Eitself (ty_2Epair_2Eprod A_27t A_27w))}) \quad (65)$$

Let $c_2Ebinary_ieee_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Ethreshold \\ A_27t A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself (ty_2Epair_2Eprod A_27t A_27w))}) \quad (66)$$

Definition 59 We define $c_Ewords_Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ c_2Ebinary_ieee_2Erounding2num : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Erounding2num \in (ty_2Enum_2Enum^{ty_2Ebinary_ieee_2Erounding}) \quad (67)$$

Definition 60 We define $c_2Ebinary_ieee_2Erounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinary_ieee_2E$

Definition 61 We define $c_2Ebinary_ieee_2ERound$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary_ie$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)})) \quad (68)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)})) \quad (69)$$

Definition 62 We define $c_2Ebinary_ieee_2Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinar$

Definition 63 We define $c_2Ebinary_ieee_2Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebin$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MIN\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (70)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (71)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (72)$$

Definition 64 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).$

Definition 65 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 66 We define $c_2Ewords_2Enzcv$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b \in ($

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (73)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (74)$$

Definition 67 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 68 We define $c_2Ewords_2Eword_ls$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1b$

Definition 69 We define $c_2Ebinary_ieee_2Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebin$

Definition 70 We define $c_2Ebinary_ieee_2Efloat_round_with_flags$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode$

Definition 71 We define $c_2Ebinary_ieee_2Efloat_some_qnan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0fp_op \in (ty$

Definition 72 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair$

Definition 73 We define $c_2Ebinary_ieee_2Efloat_add$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary$

Let $c_2Ebinary_ieee_2Efloat_minus_min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_min \\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)}) \end{aligned} \quad (75)$$

Let $c_2Ebinary_ieee_2Efloat_plus_min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_min \\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)}) \end{aligned} \quad (76)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0a \in ty_2Erealx_2Ereal. \\ (\forall V1f \in (A_27a^{ty_2Erealx_2Ereal}).(\forall V2v \in A_27a. \\ (\forall V3v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\ A_27a)\ (ap\ c_2Ebinary_ieee_2Efloat\ V0a))\ V1f)\ V2v)\ V3v1) = (ap \\ V1f\ V0a)))))) \wedge ((\forall V4f \in (A_27a^{ty_2Erealx_2Ereal}).(\forall V5v \in \\ A_27a.(\forall V6v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\ A_27a)\ c_2Ebinary_ieee_2EInfinity)\ V4f)\ V5v)\ V6v1) = V5v)))) \wedge \\ (\forall V7f \in (A_27a^{ty_2Erealx_2Ereal}).(\forall V8v \in A_27a. \\ (\forall V9v1 \in A_27a.((ap\ (ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_value_CASE \\ A_27a)\ c_2Ebinary_ieee_2ENaN)\ V7f)\ V8v)\ V9v1) = V9v1)))))) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27t. \\
& nonempty\ A.27t \Rightarrow \forall A.27w.nonempty\ A.27w \Rightarrow (((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_plus_infinity \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = c.2Ebinary_ieee.2EInfinity) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_minus_infinity \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = c.2Ebinary_ieee.2EInfinity) \wedge ((\forall V0fp_op \in \\
& \quad (ty.2Ebinary_ieee.2Efp_op\ A.27a\ A.27b).((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27a\ A.27b)\ (ap\ (c.2Ebinary_ieee.2Efloat_some_qnan\ A.27a \\
& \quad A.27b)\ V0fp_op)) = c.2Ebinary_ieee.2ENaN) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_plus_zero\ A.27t \\
& \quad A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = \\
& \quad (ap\ c.2Ebinary_ieee.2EFloat\ (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))) \wedge \\
& \quad (((ap\ (c.2Ebinary_ieee.2Efloat_value\ A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_minus_zero \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = (ap\ c.2Ebinary_ieee.2EFloat\ (ap\ c.2Ereal.2Ereal_of_num \\
& \quad c.2Enum.2E0))) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value\ A.27t \\
& \quad A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_plus_min\ A.27t\ A.27w) \\
& \quad (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = (\\
& \quad ap\ c.2Ebinary_ieee.2EFloat\ (ap\ (ap\ c.2Ereal.2E.2F\ (ap\ c.2Ereal.2Ereal_of_num \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\
& \quad (ap\ (ap\ c.2Ereal.2Epow\ (ap\ c.2Ereal.2Ereal_of_num\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (ap \\
& \quad c.2Earithmetic.2E.2B\ (ap\ (c.2Ewords.2EINT_MAX\ A.27w)\ (c.2Ebool.2Ethe_value \\
& \quad A.27w)))\ (ap\ (c.2Efc.2Edimindex\ A.27t)\ (c.2Ebool.2Ethe_value \\
& \quad A.27t)))))) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value\ A.27t\ A.27w) \\
& \quad (ap\ (c.2Ebinary_ieee.2Efloat_minus_min\ A.27t\ A.27w)\ (c.2Ebool.2Ethe_value \\
& \quad (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = (ap\ c.2Ebinary_ieee.2EFloat \\
& \quad (ap\ (ap\ c.2Ereal.2E.2F\ (ap\ c.2Ereal.2Ereal_neg\ (ap\ c.2Ereal.2Ereal_of_num \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\
& \quad (ap\ (ap\ c.2Ereal.2Epow\ (ap\ c.2Ereal.2Ereal_of_num\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (ap \\
& \quad c.2Earithmetic.2E.2B\ (ap\ (c.2Ewords.2EINT_MAX\ A.27w)\ (c.2Ebool.2Ethe_value \\
& \quad A.27w)))\ (ap\ (c.2Efc.2Edimindex\ A.27t)\ (c.2Ebool.2Ethe_value \\
& \quad A.27t)))))))))
\end{aligned} \tag{78}$$

Assume the following.

$$True \tag{79}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\
A.27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((\\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (82)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& True)) \quad (83)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (84)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (85)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x_27 \in 2.(\forall V2y \in 2.(\forall V3y_27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))))) \quad (86)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2. \\
& (\forall V2x \in A_27a.(\forall V3x_27 \in A_27a.(\forall V4y \in A_27a. \\
& (\forall V5y_27 \in A_27a.(((p \ V0P) \Leftrightarrow (p \ V1Q)) \wedge ((p \ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\
& ((\neg(p \ V1Q)) \Rightarrow (V4y = V5y_27)))) \Rightarrow ((ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \\
& V0P) \ V2x) \ V4y) = (ap \ (ap \ (ap \ (c_2Ebool_2ECOND \ A_27a) \ V1Q) \ V3x_27) \\
& V5y_27)))))) \quad (87)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\
& \forall V0x \in A_27a.(\forall V1y \in A_27b.(\forall V2a \in A_27a.(\forall V3b \in \\
& A_27b.(((ap \ (ap \ (c_2Epair_2E_2C \ A_27a \ A_27b) \ V0x) \ V1y) = (ap \ (ap \\
& (c_2Epair_2E_2C \ A_27a \ A_27b) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (88)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow \forall A_{.27b}.nonempty\ A_{.27b} \Rightarrow \forall A_{.27c}. \\
& \quad nonempty\ A_{.27c} \Rightarrow (\forall V0x \in A_{.27b}. (\forall V1y \in A_{.27c}. (\forall V2f \in \\
& \quad ((A_{.27a}^{A_{.27c}})^{A_{.27b}}). ((ap\ (ap\ (c_2Epair_2Epair_CASE\ A_{.27a}\ A_{.27b} \\
& \quad A_{.27c})\ (ap\ (ap\ (c_2Epair_2E_2C\ A_{.27b}\ A_{.27c})\ V0x)\ V1y))\ V2f) = (ap \\
& \quad (ap\ V2f\ V0x)\ V1y))))))
\end{aligned} \tag{89}$$

Theorem 1

$$\begin{aligned}
& \forall A_{.27t}.nonempty\ A_{.27t} \Rightarrow \forall A_{.27w}.nonempty\ A_{.27w} \Rightarrow (\\
& \quad \forall V0mode \in ty_2Ebinary_ieee_2Erounding. (\forall V1x \in \\
& \quad (ty_2Ebinary_ieee_2Efloat\ A_{.27t}\ A_{.27w}). (\forall V2r \in ty_2Erealax_2Ereal. \\
& \quad ((ap\ (c_2Ebinary_ieee_2Efloat_value\ A_{.27t}\ A_{.27w})\ V1x) = (ap \\
& \quad c_2Ebinary_ieee_2Efloat\ V2r)) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_add \\
& \quad A_{.27t}\ A_{.27w})\ V0mode)\ V1x)\ (ap\ (c_2Ebinary_ieee_2Efloat_plus_infinity \\
& \quad A_{.27t}\ A_{.27w}))\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_{.27t} \\
& \quad A_{.27w})))) = (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Ebinary_ieee_2Eflags \\
& \quad (ty_2Ebinary_ieee_2Efloat\ A_{.27t}\ A_{.27w}))\ c_2Ebinary_ieee_2Eclear_flags) \\
& \quad (ap\ (c_2Ebinary_ieee_2Efloat_plus_infinity\ A_{.27t}\ A_{.27w}) \\
& \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_{.27t}\ A_{.27w}))))))
\end{aligned}$$