

thm_2Ebinary_ieee_2Efloat_div_finite_plus_infinity
(TMMXrCmvB-
sqJ2YZkxDqxr4LaM5j1qp93CFD)

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Let $ty_2Efc_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Efc_2Ecart\ A0\ A1) \quad (1)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (2)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (3)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (4)$$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (5)$$

Let $c_2Ebinary_ieee_2Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand_fupd\ A_27t\ A_27u\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (6)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x. \\ & \quad nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w\ A_27x \in \\ & \quad ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{((ty_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})} \end{aligned} \quad (7)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (8)$$

Let $c_2Ebinary_ieee_2Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign_fupd\ A_27t\ A_27w \in \\ & \quad ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{((ty_2Efloat\ A_27t\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})} \end{aligned} \quad (9)$$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (10)$$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (11)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (12)$$

Let $ty_2Ebinary_ieee_2Eflags : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Eflags \quad (13)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$.

Definition 3 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x)))$.

Definition 4 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$.

Let $c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_AfterRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (14)$$

Let $c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Underflow_BeforeRounding_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (15)$$

Let $c_2Ebinary_ieee_2Eflags_Precision_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Precision_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (16)$$

Let $c_2Ebinary_ieee_2Eflags_Overflow_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_Overflow_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (17)$$

Let $c_2Ebinary_ieee_2Eflags_InvalidOp_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_InvalidOp_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (18)$$

Let $c_2Ebinary_ieee_2Eflags_DivideByZero_fupd : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Eflags_DivideByZero_fupd \in ((ty_2Ebinary_ieee_2Eflags^{ty_2Ebinary_ieee_2Eflags})^{(2^2)}) \quad (19)$$

Definition 6 We define $c_2Ebinary_ieee_2Eclear_flags$ to be $(ap (ap c_2Ebinary_ieee_2Eflags_DivideByZero_fupd))$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (20)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (21)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (22)$$

Definition 7 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 8 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (23)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (24)$$

Definition 9 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (25)$$

Definition 10 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 11 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (26)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (27)$$

Let $c_2Efcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcp_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (28)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (29)$$

Let $ty_2Efcp_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcp_2Efinite_image\ A0) \quad (30)$$

Definition 12 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 13 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E$

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2$

Definition 15 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A)\ \lambda y$ of type $\iota \Rightarrow \iota$.

Definition 16 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 17 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 18 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ c_2Ebool_2E_2F_5C$

Definition 19 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota.(ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum$

Let $c_2Efc_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efc_2Edest_cart \\ & A_27a\ A_27b \in ((A_27a^{(ty_2Efc_2Efinite_image\ A_27b)})^{(ty_2Efc_2Ecart\ A_27a\ A_27b)}) \end{aligned} \quad (31)$$

Definition 20 We define $c_2Efc_2Efc_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efc_2Ecart\ A_27a\ A_27b).$

Definition 21 We define $c_2Ewords_2Eword_msb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (ap\ (c_2Efc_2Efc_index\ w)))$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (32)$$

Definition 22 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT2\ n))$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (33)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (34)$$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (35)$$

Definition 23 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_2Ebool_2ECOND\ t1\ t2\ t3))))$

Definition 24 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ b\ n))))$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (36)$$

Definition 25 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a).(ap\ (ap\ (c_2Ewords_2Eword_msb\ w)))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (37)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod \\ & A0\ A1) \end{aligned} \quad (38)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (39)$$

Definition 26 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (t$

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (40)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (41)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}} \quad (42)$$

Definition 27 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty$

Definition 28 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (43)$$

Definition 29 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Definition 30 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Let $c_2Erealax_2Etreal_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) \quad (44)$$

Definition 31 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Ewords_2EINT_MAX A_27a \in (ty_2Enum_2Enum)^{(ty_2Ebool_2Eitself A_27a)} \quad (45)$$

Let $c_2Ebinary_ieee_2Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Sign A_27t A_27w \in ((ty_2EfcP_2Ecart 2 ty_2Eone_2Eone)^{(ty_2Ebinary_ieee_2Efloat A_27t A_27w)}) \quad (46)$$

Let $c_2Erealax_2Etreal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (47)$$

Definition 32 We define `c_2Erealax_2Ereal_neg` to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Let `c_2Earithmetic_2EDIV` : ι be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (48)$$

Definition 33 We define `c_2Ebit_2EDIV_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let `c_2Earithmetic_2EMOD` : ι be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (49)$$

Definition 34 We define `c_2Ebit_2EMOD_2EXP` to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 35 We define `c_2Ebit_2EBITS` to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 36 We define `c_2Ebit_2EBIT` to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 37 We define `c_2Efcg_2EFCG` to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 38 We define `c_2Ewords_2En2w` to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap\ (c_2Efcg_2EFCG$

Definition 39 We define `c_2Ebinary_ieee_2Efloat_to_real` to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let `ty_2Ebinaary_ieee_2Efloat_value` : ι be given. Assume the following.

$$nonempty\ ty_2Ebinaary_ieee_2Efloat_value \quad (50)$$

Let `c_2Ebinaary_ieee_2EFloat` : ι be given. Assume the following.

$$c_2Ebinaary_ieee_2EFloat \in (ty_2Ebinaary_ieee_2Efloat_value^{ty_2Erealax_2Ereal}) \quad (51)$$

Let `c_2Ebinaary_ieee_2ENaN` : ι be given. Assume the following.

$$c_2Ebinaary_ieee_2ENaN \in ty_2Ebinaary_ieee_2Efloat_value \quad (52)$$

Let `c_2Ebinaary_ieee_2EInfinity` : ι be given. Assume the following.

$$c_2Ebinaary_ieee_2EInfinity \in ty_2Ebinaary_ieee_2Efloat_value \quad (53)$$

Let `c_2Ewords_2EUINT_MAX` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (\quad (54)$$

$$ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)})$$

Definition 40 We define `c_2Ewords_2Eword_T` to be $\lambda A_27a : \iota.(ap\ (c_2Ewords_2En2w\ A_27a)\ (ap\ (c_2Ew$

Definition 41 We define `c_2Ebinaary_ieee_2Efloat_value` to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Let $c_2Ebinary_ieee_2Efloat_value_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebinary_ieee_2Efloat_value_CASE\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebinary_ieee_2Efloat_value}) \quad (55)$$

Definition 42 We define $c_2Ebinary_ieee_2Efloat_is_nan$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value_CASE\ A_27t\ A_27w)$

Definition 43 We define $c_2Ebinary_ieee_2Efloat_is_signalling$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value_CASE\ A_27t\ A_27w)$

Let $c_2Elist_2EEXISTS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EEXISTS\ A_27a \in ((2^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (56)$$

Definition 44 We define $c_2Ebinary_ieee_2Echeck_for_signalling$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0l \in (ty_2Ebinary_ieee_2Efloat_value_CASE\ A_27a\ A_27b)$

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (57)$$

Definition 45 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 46 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal.$

Definition 47 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal. (ap\ (ap\ (ap\ (c_2Ebool_2ECONJ\ x\ y)\ z)\ w)\ v)$

Let $c_2Ebinary_ieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Elargest\ A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \quad (58)$$

Definition 48 We define $c_2Ebinary_ieee_2Efloat_is_finite$ to be $\lambda A_27t : \iota. \lambda A_27w : \iota. \lambda V0x \in (ty_2Ebinary_ieee_2Efloat_value_CASE\ A_27t\ A_27w)$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (59)$$

Definition 49 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Ebool_2ECONJ\ x\ y)\ z)$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (60)$$

Definition 50 We define $c_Ereal_Ereal_sub$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Definition 51 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 52 We define $c_EBinary_ieee_Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_EBinary_iee)})$

Definition 53 We define $c_EBinary_ieee_EClosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_EBinary_iee)})$

Definition 54 We define $c_EBinary_ieee_EClosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_EBinary_ieee_EClosest_such\ A_27a\ A_27b))$

Let $c_EBinary_ieee_Efloat_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_EBinary_ieee_Efloat_top \\ & A_27t\ A_27w \in ((ty_EBinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (61)$$

Definition 55 We define $c_Ereal_Ereal_gt$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_EBinary_ieee_Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_EBinary_ieee_Efloat_bottom \\ & A_27t\ A_27w \in ((ty_EBinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (62)$$

Definition 56 We define c_Ebool_ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27a$

Let $c_EBinary_ieee_Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_EBinary_ieee_Efloat_minus_infinity \\ & A_27t\ A_27w \in ((ty_EBinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (63)$$

Definition 57 We define $c_Ereal_Ereal_ge$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_EBinary_ieee_Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_EBinary_ieee_Efloat_plus_infinity \\ & A_27t\ A_27w \in ((ty_EBinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (64)$$

Let $c_EBinary_ieee_Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_EBinary_ieee_Ethreshold \\ & A_27t\ A_27w \in (ty_Erealax_Ereal^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (65)$$

Definition 58 We define $c_Ewords_Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efcpcart\ 2\ A_27a).(ap$

Let $ty_2Ebinary_ieee_2Erunding : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Erunding \quad (66)$$

Let $c_2Ebinary_ieee_2Erunding2num : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Erunding2num \in (ty_2Enum_2Enum^{ty_2Ebinary_ieee_2Erunding}) \quad (67)$$

Definition 59 We define $c_2Ebinary_ieee_2Erunding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinary_ieee_2Erunding$

Definition 60 We define $c_2Ebinary_ieee_2Erund$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary_ieee_2Erunding$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (68)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (69)$$

Definition 61 We define $c_2Ebinary_ieee_2Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary_ieee_2Erunding)$

Definition 62 We define $c_2Ebinary_ieee_2Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary_ieee_2Erunding$

Let $c_2Ewords_2EINT_MIN : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MIN\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (70)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (71)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2Edimword\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (72)$$

Definition 63 We define $c_2Ewords_2Eword_2comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efc_2Ecart\ 2\ A_27a)$.

Definition 64 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Definition 65 We define $c_2Ewords_2Enzcv$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Efc_2Ecart\ 2\ A_27a).\lambda V1b \in (ty_2Efc_2Ecart\ 2\ A_27a)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (73)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (74)$$

Definition 66 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})$

Definition 67 We define $c_2Ewords_2Eword_ls$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2EfcP_2Ecart\ 2\ A_27a).\lambda V1b$

Definition 68 We define $c_2Ebinary_ieee_2Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina$

Definition 69 We define $c_2Ebinary_ieee_2Efloat_round_with_flags$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode$

Definition 70 We define $c_2Ebinary_ieee_2Edividezero_flags$ to be $(ap\ (ap\ c_2Ebina$

Let ty_2Ebina

$$\begin{aligned} \forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebina_ieee_2Efp_op \\ A0\ A1) \end{aligned} \quad (75)$$

Let c_2Ebina

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebina \\ A_27t\ A_27w \in (((ty_2Ebina_ieee_2Efp_op\ A_27t\ A_27w)^{(ty_2Ebina_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Eb$$
 \end{aligned} \quad (76)

Definition 71 We define c_2Ebina

Definition 72 We define c_2Ebina

Definition 73 We define $c_2Epair_2Epair_CASE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0p \in (ty_2Epair$

Definition 74 We define c_2Ebina

Let c_2Ebina

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebina_ieee_2Efloat_minus_min \\ A_27t\ A_27w \in ((ty_2Ebina_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))} \end{aligned} \quad (77)$$

Let $c_2Ebinary_ieee_2Efloat_plus_min : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_min \\ & A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w)} \\ & \hspace{15em} (78) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27t.\text{nonempty } A_27t \Rightarrow \forall A_27u.\text{nonempty } A_27u \Rightarrow \forall A_27w. \\
& \quad \text{nonempty } A_27w \Rightarrow \forall A_27x.\text{nonempty } A_27x \Rightarrow ((\forall V0f0 \in \\
& \quad ((ty_2EfcP_2Ecart\ 2\ A_27x)^{(ty_2EfcP_2Ecart\ 2\ A_27w)}).(\forall V1f \in \\
& \quad (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_27t\ A_27x)\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd \\
& \quad A_27t\ A_27w\ A_27x)\ V0f0)\ V1f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_27t\ A_27w)\ V1f)))) \wedge ((\forall V2f0 \in ((ty_2EfcP_2Ecart\ 2\ A_27u)^{(ty_2EfcP_2Ecart\ 2\ A_27t)}). \\
& \quad (\forall V3f \in (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign\ A_27u\ A_27w)\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_27t\ A_27u\ A_27w)\ V2f0)\ V3f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_27t\ A_27w)\ V3f)))) \wedge ((\forall V4f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}). \\
& \quad (\forall V5f \in (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_27t\ A_27w)\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_27t\ A_27w)\ V4f0)\ V5f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_27t\ A_27w)\ V5f)))) \wedge ((\forall V6f0 \in ((ty_2EfcP_2Ecart\ 2\ A_27u)^{(ty_2EfcP_2Ecart\ 2\ A_27t)}). \\
& \quad (\forall V7f \in (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_27u\ A_27w)\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_27t\ A_27u\ A_27w)\ V6f0)\ V7f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_27t\ A_27w)\ V7f)))) \wedge ((\forall V8f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}). \\
& \quad (\forall V9f \in (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Significand\ A_27t\ A_27w)\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign_fupd\ A_27t\ A_27w)\ V8f0)\ V9f)) = \\
& \quad (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_27t\ A_27w)\ V9f)))) \wedge \\
& \quad ((\forall V10f0 \in ((ty_2EfcP_2Ecart\ 2\ A_27x)^{(ty_2EfcP_2Ecart\ 2\ A_27w)}). \\
& \quad (\forall V11f \in (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (\\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_27t\ A_27x)\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w\ A_27x)\ \\
& \quad V10f0)\ V11f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_27t \\
& \quad A_27w)\ V11f)))) \wedge ((\forall V12f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}). \\
& \quad (\forall V13f \in (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (\\
& \quad (c_2EbinaRy_ieee_2Efloat_Sign\ A_27t\ A_27w)\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_27t\ A_27w)\ V12f0)\ V13f)) = (ap\ V12f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_27t\ A_27w)\ V13f)))) \wedge ((\forall V14f0 \in ((ty_2EfcP_2Ecart\ 2 \\
& \quad A_27x)^{(ty_2EfcP_2Ecart\ 2\ A_27w)}).(\forall V15f \in (ty_2EbinaRy_ieee_2Efloat \\
& \quad A_27t\ A_27w).((ap\ (c_2EbinaRy_ieee_2Efloat_Exponent\ A_27t \\
& \quad A_27x)\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_27t \\
& \quad A_27w\ A_27x)\ V14f0)\ V15f)) = (ap\ V14f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_27t\ A_27w)\ V15f)))) \wedge ((\forall V16f0 \in ((ty_2EfcP_2Ecart\ 2\ A_27u)^{(ty_2EfcP_2Ecart\ 2\ A_27t)}). \\
& \quad (\forall V17f \in (ty_2EbinaRy_ieee_2Efloat\ A_27t\ A_27w).((ap\ (\\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_27u\ A_27w)\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Significand_fupd\ A_27t\ A_27u\ A_27w)\ \\
& \quad V16f0)\ V17f)) = (ap\ V16f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand \\
& \quad A_27t\ A_27w)\ V17f)))))))))))))
\end{aligned}
\tag{79}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.\text{nonempty } A_27a \Rightarrow ((\forall V0a \in \text{ty_2Erealax_2Ereal}.) \\
& \quad (\forall V1f \in (A_27a^{\text{ty_2Erealax_2Ereal}}).(\forall V2v \in A_27a. \\
& \quad (\forall V3v1 \in A_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_value_CASE} \\
& \quad A_27a) (\text{ap } \text{c_2Ebinary_ieee_2Efloat } V0a)) V1f) V2v) V3v1) = (\text{ap} \\
& \quad V1f V0a)))))) \wedge ((\forall V4f \in (A_27a^{\text{ty_2Erealax_2Ereal}}).(\forall V5v \in \\
& A_27a.(\forall V6v1 \in A_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_value_CASE} \\
& \quad A_27a) \text{c_2Ebinary_ieee_2EInfinity}) V4f) V5v) V6v1) = V5v)))) \wedge \\
& \quad (\forall V7f \in (A_27a^{\text{ty_2Erealax_2Ereal}}).(\forall V8v \in A_27a. \\
& \quad (\forall V9v1 \in A_27a.((\text{ap } (\text{ap } (\text{ap } (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_value_CASE} \\
& \quad A_27a) \text{c_2Ebinary_ieee_2ENaN}) V7f) V8v) V9v1) = V9v1)))))) \\
& \hspace{10em} (80)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27t.\text{nonempty } A_27t \Rightarrow \forall A_27w.\text{nonempty } A_27w \Rightarrow (\\
& \quad (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_plus_infinity } A_27t A_27w) \\
& \quad (\text{c_2Ebool_2Ethe_value } (\text{ty_2Epair_2Eprod } A_27t A_27w))) = (\text{ap} \\
& \quad (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_Sign_fupd } A_27t A_27w) (\text{ap } (\\
& \quad \text{c_2Ecombin_2EK } (\text{ty_2Efc_2Ecart } 2 \text{ ty_2Eone_2Eone}) (\text{ty_2Efc_2Ecart} \\
& \quad 2 \text{ ty_2Eone_2Eone})) (\text{ap } (\text{c_2Ewords_2En2w } \text{ty_2Eone_2Eone}) \text{c_2Enum_2E0}))) \\
& \quad (\text{ap } (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_Exponent_fupd } A_27t A_27w \\
& \quad A_27w) (\text{ap } (\text{c_2Ecombin_2EK } (\text{ty_2Efc_2Ecart } 2 A_27w) (\text{ty_2Efc_2Ecart} \\
& \quad 2 A_27w)) (\text{c_2Ewords_2Eword_T } A_27w))) (\text{ap } (\text{ap } (\text{c_2Ebinary_ieee_2Efloat_Significand_fupd} \\
& \quad A_27t A_27t A_27w) (\text{ap } (\text{c_2Ecombin_2EK } (\text{ty_2Efc_2Ecart } 2 A_27t) \\
& \quad (\text{ty_2Efc_2Ecart } 2 A_27t)) (\text{ap } (\text{c_2Ewords_2En2w } A_27t) \text{c_2Enum_2E0}))) \\
& \quad (\text{c_2Ebool_2EARB } (\text{ty_2Ebinary_ieee_2Efloat } A_27t A_27w)))))) \\
& \hspace{10em} (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27t. \\
& nonempty\ A.27t \Rightarrow \forall A.27w.nonempty\ A.27w \Rightarrow (((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_plus_infinity \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = c.2Ebinary_ieee.2EInfinity) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_minus_infinity \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = c.2Ebinary_ieee.2EInfinity) \wedge ((\forall V0fp_op \in \\
& \quad (ty.2Ebinary_ieee.2Efp_op\ A.27a\ A.27b).((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27a\ A.27b)\ (ap\ (c.2Ebinary_ieee.2Efloat_some_qnan\ A.27a \\
& \quad A.27b)\ V0fp_op)) = c.2Ebinary_ieee.2ENaN) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_plus_zero\ A.27t \\
& \quad A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = \\
& \quad (ap\ c.2Ebinary_ieee.2EFloat\ (ap\ c.2Ereal.2Ereal_of_num\ c.2Enum.2E0))) \wedge \\
& \quad (((ap\ (c.2Ebinary_ieee.2Efloat_value\ A.27t\ A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_minus_zero \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = (ap\ c.2Ebinary_ieee.2EFloat\ (ap\ c.2Ereal.2Ereal_of_num \\
& \quad c.2Enum.2E0))) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value\ A.27t \\
& \quad A.27w)\ (ap\ (c.2Ebinary_ieee.2Efloat_plus_min\ A.27t\ A.27w) \\
& \quad (c.2Ebool.2Ethe_value\ (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = (\\
& \quad ap\ c.2Ebinary_ieee.2EFloat\ (ap\ (ap\ c.2Ereal.2E.2F\ (ap\ c.2Ereal.2Ereal_of_num \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\
& \quad (ap\ (ap\ c.2Ereal.2Epow\ (ap\ c.2Ereal.2Ereal_of_num\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (ap \\
& \quad c.2Earithmetic.2E.2B\ (ap\ (c.2Ewords.2EINT_MAX\ A.27w)\ (c.2Ebool.2Ethe_value \\
& \quad A.27w)))\ (ap\ (c.2Efc.2Edimindex\ A.27t)\ (c.2Ebool.2Ethe_value \\
& \quad A.27t)))))) \wedge (((ap\ (c.2Ebinary_ieee.2Efloat_value\ A.27t\ A.27w) \\
& \quad (ap\ (c.2Ebinary_ieee.2Efloat_minus_min\ A.27t\ A.27w)\ (c.2Ebool.2Ethe_value \\
& \quad (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = (ap\ c.2Ebinary_ieee.2EFloat \\
& \quad (ap\ (ap\ c.2Ereal.2E.2F\ (ap\ c.2Ereal.2Ereal_neg\ (ap\ c.2Ereal.2Ereal_of_num \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\
& \quad (ap\ (ap\ c.2Ereal.2Epow\ (ap\ c.2Ereal.2Ereal_of_num\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (ap \\
& \quad c.2Earithmetic.2E.2B\ (ap\ (c.2Ewords.2EINT_MAX\ A.27w)\ (c.2Ebool.2Ethe_value \\
& \quad A.27w)))\ (ap\ (c.2Efc.2Edimindex\ A.27t)\ (c.2Ebool.2Ethe_value \\
& \quad A.27t)))))))))
\end{aligned} \tag{82}$$

Assume the following.

$$True \tag{83}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{84}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (85)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (86)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (87)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (88)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (89)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (90)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (91)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (92)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. (((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2ET) V0t1) \\ & V1t2) = V0t1) \wedge ((ap (ap (ap (c_2Ebool_2ECOND A_27a) c_2Ebool_2EF) \\ & V0t1) V1t2) = V1t2)))) \end{aligned} \quad (93)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (94)$$

Assume the following.

$$2.(((\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \Rightarrow (95)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.((ap (ap (c_{.2}Ecombin_{.2}EK A_{.27a} A_{.27b}) V0x) V1y) = V0x))) \quad (96)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\forall V0x \in A_{.27a}.(\forall V1y \in A_{.27b}.(\forall V2a \in A_{.27a}.(\forall V3b \in A_{.27b}.(((ap (ap (c_{.2}Epair_{.2}E_{.2}C A_{.27a} A_{.27b}) V0x) V1y) = (ap (ap (c_{.2}Epair_{.2}E_{.2}C A_{.27a} A_{.27b}) V2a) V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \Rightarrow (97)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow \forall A_{.27c}.nonempty A_{.27c} \Rightarrow (\forall V0x \in A_{.27b}.(\forall V1y \in A_{.27c}.(\forall V2f \in (A_{.27a}^{A_{.27c}})^{A_{.27b}}).((ap (ap (c_{.2}Epair_{.2}Epair_{.2}CASE A_{.27a} A_{.27b} A_{.27c}) (ap (ap (c_{.2}Epair_{.2}E_{.2}C A_{.27b} A_{.27c}) V0x) V1y)) V2f) = (ap (ap V2f V0x) V1y)))))) \Rightarrow (98)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0m \in ty_{.2}Enum_{.2}Enum_{.2}.(\forall V1n \in ty_{.2}Enum_{.2}Enum_{.2}.(((ap (c_{.2}Ewords_{.2}En2w A_{.27a}) V0m) = (ap (c_{.2}Ewords_{.2}En2w A_{.27a}) V1n)) \Leftrightarrow ((ap (ap c_{.2}Earithmetic_{.2}EMOD V0m) (ap (c_{.2}Ewords_{.2}Edimword A_{.27a}) (c_{.2}Ebool_{.2}Ethe_{.2}value A_{.27a}))) = (ap (ap c_{.2}Earithmetic_{.2}EMOD V1n) (ap (c_{.2}Ewords_{.2}Edimword A_{.27a}) (c_{.2}Ebool_{.2}Ethe_{.2}value A_{.27a})))))) \Rightarrow (99)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow ((ap (c_{.2}Ewords_{.2}Eword_{.2}comp A_{.27a}) (ap (c_{.2}Ewords_{.2}En2w A_{.27a}) (ap c_{.2}Earithmetic_{.2}ENUMERAL (ap c_{.2}Earithmetic_{.2}EBIT1 c_{.2}Earithmetic_{.2}EZERO)))) = (c_{.2}Ewords_{.2}Eword_{.2}T A_{.27a})) \quad (100)$$

Theorem 1
$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\ & \quad \forall V0mode \in ty_2Ebinary_ieee_2Erunding. (\forall V1x \in \\ & (ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w). (\forall V2r \in ty_2Erealax_2Ereal. \\ & \quad ((ap\ (c_2Ebinary_ieee_2Efloat_value\ A_27t\ A_27w)\ V1x) = (ap \\ & \quad c_2Ebinary_ieee_2Efloat\ V2r)) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_div \\ & \quad A_27t\ A_27w)\ V0mode)\ V1x)\ (ap\ (c_2Ebinary_ieee_2Efloat_plus_infinity \\ & \quad A_27t\ A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t \\ & \quad A_27w)))) = (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Ebinary_ieee_2Eflags \\ & \quad (ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w))\ c_2Ebinary_ieee_2Eclear_flags) \\ & \quad (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ (ty_2Ebinary_ieee_2Efloat\ A_27t \\ & \quad A_27w))\ (ap\ (ap\ (c_2Emin_2E_3D\ (ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)) \\ & \quad (ap\ (c_2Ebinary_ieee_2Efloat_Sign\ A_27t\ A_27w)\ V1x))\ (ap\ (c_2Ewords_2En2w \\ & \quad ty_2Eone_2Eone)\ c_2Enum_2E0)))\ (ap\ (c_2Ebinary_ieee_2Efloat_plus_zero \\ & \quad A_27t\ A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t \\ & \quad A_27w))))\ (ap\ (c_2Ebinary_ieee_2Efloat_minus_zero\ A_27t \\ & \quad A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t\ A_27w))))))))) \end{aligned}$$