

thm\_2Ebinary\_2Eieee\_2Efloat\_fn\_updates  
(TMQxQgE36AzowR7ph95j2evkzRpT1niXQ7m)

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**Definition 1** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E2T$  to be  $(ap (ap (c\_2Emin\_2E3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2) (\lambda V2t \in 2.V2t)))$

Let  $ty\_2Ebinary\_2Eieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_2Eieee\_2Efloat A0 A1) \tag{1}$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty ty\_2Eone\_2Eone \tag{2}$$

Let  $ty\_2Efcf\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Efcf\_2Ecart A0 A1) \tag{3}$$

Let  $c\_2Ebinary\_2Eieee\_2Efloat\_2ESign\_2Eupd : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.\lambda t.nonempty A.\lambda w.nonempty A.\lambda w \Rightarrow c\_2Ebinary\_2Eieee\_2Efloat A.t A.w \in (((ty\_2Ebinary\_2Eieee\_2Efloat A.t A.w)(ty\_2Ebinary\_2Eieee\_2Efloat A.t A.w))((ty\_2Ebinary\_2Eieee\_2Efloat A.t A.w))) \tag{4}$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x. \\ & \quad nonempty\ A\_27x \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd\ A\_27t\ A\_27w\ A\_27x \in (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}))^{(ty\_2Efloat\ A\_27t\ A\_27w)} \\ & \hspace{15em} (5) \end{aligned}$$

Let  $c\_2Ebinary\_ieee\_2Erecordtype\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Erecordtype\_2Efloat\ A\_27t\ A\_27w \in (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27w)})^{(ty\_2Efloat\ A\_27t\ A\_27w)} \\ & \hspace{15em} (6) \end{aligned}$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27w. \\ & \quad nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd\ A\_27t\ A\_27u\ A\_27w \in (((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27u)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27u)}))^{(ty\_2Efloat\ A\_27t\ A\_27u)} \\ & \hspace{15em} (7) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ( \\ & \quad \forall V0f \in ((ty\_2Efloat\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efloat\_2Ecart\ 2\ ty\_2Eone\_2Eone)}), \\ & \quad (\forall V1c \in (ty\_2Efloat\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V2c0 \in \\ & \quad (ty\_2Efloat\_2Ecart\ 2\ A\_27w).(\forall V3c1 \in (ty\_2Efloat\_2Ecart\ 2 \\ & \quad A\_27t).((ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_Significand\_fupd\ A\_27t \\ & \quad A\_27w)\ V0f)\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Erecordtype\_2Efloat \\ & \quad A\_27t\ A\_27w)\ V1c)\ V2c0)\ V3c1))) = (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Erecordtype\_2Efloat \\ & \quad A\_27t\ A\_27w)\ (ap\ V0f\ V1c))\ V2c0)\ V3c1)))))) \\ & \hspace{15em} (8) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x. \\ & \quad nonempty\ A\_27x \Rightarrow (\forall V0f \in ((ty\_2Efloat\_2Ecart\ 2\ A\_27x)^{(ty\_2Efloat\_2Ecart\ 2\ A\_27x)}), \\ & \quad (\forall V1c \in (ty\_2Efloat\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V2c0 \in \\ & \quad (ty\_2Efloat\_2Ecart\ 2\ A\_27w).(\forall V3c1 \in (ty\_2Efloat\_2Ecart\ 2 \\ & \quad A\_27t).((ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_Exponent\_fupd\ A\_27t\ A\_27w\ A\_27x)\ V0f)\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Erecordtype\_2Efloat \\ & \quad A\_27t\ A\_27w)\ V1c)\ V2c0)\ V3c1))) = (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Erecordtype\_2Efloat \\ & \quad A\_27t\ A\_27x)\ V1c)\ (ap\ V0f\ V2c0))\ V3c1)))))) \\ & \hspace{15em} (9) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27t.\text{nonempty } A\_27t \Rightarrow \forall A\_27u.\text{nonempty } A\_27u \Rightarrow \forall A\_27w. \\
& \text{nonempty } A\_27w \Rightarrow (\forall V0f \in ((\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27u)^{(\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27t)}). \\
& (\forall V1c \in (\text{ty\_2Efc}p\_2Ecart\ 2\ \text{ty\_2Eone\_2Eone}).(\forall V2c0 \in \\
& (\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27w).(\forall V3c1 \in (\text{ty\_2Efc}p\_2Ecart\ 2 \\
& A\_27t).((\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Efloat\_Significand\_fupd \\
& A\_27t\ A\_27u\ A\_27w)\ V0f)\ (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27t\ A\_27w)\ V1c)\ V2c0)\ V3c1))) = (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27u\ A\_27w)\ V1c)\ V2c0)\ (\text{ap } V0f\ V3c1))))))
\end{aligned} \tag{10}$$

**Theorem 1**

$$\begin{aligned}
& \forall A\_27t.\text{nonempty } A\_27t \Rightarrow \forall A\_27u.\text{nonempty } A\_27u \Rightarrow \forall A\_27w. \\
& \text{nonempty } A\_27w \Rightarrow \forall A\_27x.\text{nonempty } A\_27x \Rightarrow ((\forall V0f \in ( \\
& (\text{ty\_2Efc}p\_2Ecart\ 2\ \text{ty\_2Eone\_2Eone})^{(\text{ty\_2Efc}p\_2Ecart\ 2\ \text{ty\_2Eone\_2Eone})}). \\
& (\forall V1c \in (\text{ty\_2Efc}p\_2Ecart\ 2\ \text{ty\_2Eone\_2Eone}).(\forall V2c0 \in \\
& (\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27w).(\forall V3c1 \in (\text{ty\_2Efc}p\_2Ecart\ 2 \\
& A\_27t).((\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Efloat\_Sign\_fupd\ A\_27t \\
& A\_27w)\ V0f)\ (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27t\ A\_27w)\ V1c)\ V2c0)\ V3c1))) = (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27t\ A\_27w)\ (\text{ap } V0f\ V1c))\ V2c0)\ V3c1)))))) \wedge ((\forall V4f \in ((\text{ty\_2Efc}p\_2Ecart \\
& 2\ A\_27x)^{(\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27w)}).(\forall V5c \in (\text{ty\_2Efc}p\_2Ecart \\
& 2\ \text{ty\_2Eone\_2Eone}).(\forall V6c0 \in (\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27w). \\
& (\forall V7c1 \in (\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27t).((\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Efloat\_Exponent\_fupd \\
& A\_27t\ A\_27w\ A\_27x)\ V4f)\ (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27t\ A\_27w)\ V5c)\ V6c0)\ V7c1))) = (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27t\ A\_27x)\ V5c)\ (\text{ap } V4f\ V6c0))\ V7c1)))))) \wedge ((\forall V8f \in ((\text{ty\_2Efc}p\_2Ecart \\
& 2\ A\_27u)^{(\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27t)}).(\forall V9c \in (\text{ty\_2Efc}p\_2Ecart \\
& 2\ \text{ty\_2Eone\_2Eone}).(\forall V10c0 \in (\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27w). \\
& (\forall V11c1 \in (\text{ty\_2Efc}p\_2Ecart\ 2\ A\_27t).((\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Efloat\_Significand\_fupd \\
& A\_27t\ A\_27u\ A\_27w)\ V8f)\ (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27t\ A\_27w)\ V9c)\ V10c0)\ V11c1))) = (\text{ap } (\text{ap } (\text{ap } (\text{c\_2Eb}inary\_ieee\_2Erecordtype\_2Efloat \\
& A\_27u\ A\_27w)\ V9c)\ V10c0)\ (\text{ap } V8f\ V11c1))))))
\end{aligned}$$