

thm\_2Ebinary\_2IEEE\_2Efloat\_is\_finite  
(TMQE9u1tzGWmSoATB3TaK1ZmJubNktVUPsh)

October 26, 2020

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \quad (1)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \quad (2)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (3)$$

Let  $c\_2EfcP\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2EfcP\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (4)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (5)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (6)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (8)$$

**Definition 4** We define  $c\_Ebool\_2ET$  to be  $(ap (ap (c\_Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap (ap (c\_Emin\_2E\_3D (2^{A\_27a}))))$

**Definition 6** We define  $c\_Eenum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap c\_2Enum\_2EABS\_num (ap (ap (c\_Emin\_2E\_3D (2^{A\_27a}))))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (9)$$

**Definition 7** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap (ap c\_2Earithmetic\_2E\_2B (ap (ap (c\_Emin\_2E\_3D (2^{A\_27a}))))$

**Definition 8** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (10)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (11)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal} \quad (12)$$

Let  $ty\_2Efcf\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcf\_2Ecart\ A0\ A1) \quad (13)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_ieee\_2Efloat\ A0\ A1) \quad (14)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t. nonempty\ A\_27t \Rightarrow \forall A\_27w. nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcf\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (15)$$

Let  $ty\_2Efcf\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcf\_2Efinite\_image\ A0) \quad (16)$$

**Definition 9** We define  $c\_Ebool\_2EF$  to be  $(ap (c\_Ebool\_2E\_21\ 2)) (\lambda V0t \in 2.V0t)$ .

**Definition 10** We define  $c\_Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2. \lambda Q \in 2. inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 11** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

**Definition 12** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

**Definition 13** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.\mathbf{if} (\exists x \in A.p (ap P x)) \mathbf{then}$  (the  $(\lambda x.x \in A \wedge P x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 14** We define `c_2Ebool_2E_3F` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40$

**Definition 15** We define `c_2Eprim_rec_2E_3C` to be  $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

**Definition 16** We define `c_2Ebool_2E_3F_21` to be  $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap c_2Ebool_2E_2F_5C$

**Definition 17** We define `c_2Efcp_2Efinite_index` to be  $\lambda A_27a : \iota.(ap (c_2Emin_2E_40 (A_27a^{ty_2Enum_2Enum}$

Let `c_2Efcp_2Edest_cart` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Efcp_2Edest\_cart \\ A_27a A_27b \in ((A_27a^{(ty_2Efcp_2Efinite\_image A_27b)})^{(ty_2Efcp_2Ecart A_27a A_27b)}) \end{aligned} \quad (17)$$

**Definition 18** We define `c_2Efcp_2Efcp_index` to be  $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2Efcp_2Ecart A_27a$

Let `c_2Earithmetic_2EEXP` :  $\iota$  be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (18)$$

**Definition 19** We define `c_2Ebool_2ECOND` to be  $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

**Definition 20** We define `c_2Ebit_2ESBIT` to be  $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap (ap (ap (c_2Ebool_2E_7E$

Let `c_2Esum_num_2ESUM` :  $\iota$  be given. Assume the following.

$$c_2Esum\_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (19)$$

**Definition 21** We define `c_2Ewords_2Ew2n` to be  $\lambda A_27a : \iota.\lambda V0w \in (ty_2Efcp_2Ecart 2 A_27a).(ap (ap$

Let `ty_2Ehreal_2Ehreal` :  $\iota$  be given. Assume the following.

$$nonempty ty_2Ehreal_2Ehreal \quad (20)$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (21)$$

Let `c_2Erealax_2Ereal_REP_CLASS` :  $\iota$  be given. Assume the following.

$$c_2Erealax_2Ereal\_REP\_CLASS \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal\_REP\_CLASS}) \quad (22)$$

**Definition 22** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (t$   
Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (23)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (24)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}})) \quad (25)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)$

**Definition 24** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (26)$$

**Definition 25** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 26** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

**Definition 27** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (27)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (28)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent A\_27t A\_27w \in ((ty\_2EfcP\_2Ecart 2 A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)}) \quad (29)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (30)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign \\ & A\_27t\ A\_27w \in ((ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \end{aligned} \quad (31)$$

Let  $c\_2Erealax\_2Etreax\_neg : \iota$  be given. Assume the following.

$$\begin{aligned} & c\_2Erealax\_2Etreax\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ & ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \end{aligned} \quad (32)$$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (33)$$

**Definition 30** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (34)$$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (35)$$

**Definition 31** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 32** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 33** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 34** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 35** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Efc\_2EFCP$

**Definition 36** We define  $c\_2Ebinary\_ieee\_2Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (36)$$

Let  $c\_2Ebinary\_ieee\_2Efloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Efloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (37)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (38)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (39)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in ( ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)} ) \quad (40)$$

**Definition 37** We define  $c\_2Ewords\_2Eword\_T$  to be  $\lambda A\_27a : \iota.(ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ (c\_2Ew$

**Definition 38** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_normal$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

**Definition 39** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_subnormal$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2E$

**Definition 40** We define  $c\_2Ebinary\_ieee\_2Efloat\_value$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let  $c\_2Ebinary\_ieee\_2Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_value\_CASE\ A\_27a \in (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_ieee\_2Efloat\_value} \quad (41)$$

**Definition 41** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_finite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

**Definition 42** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_zero$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

**Definition 43** We define  $c\_2Ebool\_2E5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V2t \in$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (42)$$

Let  $c\_2Earithmetic\_2EODD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (43)$$

**Definition 44** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

**Definition 45** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 46** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 47** We define  $c\_2Eprim\_rec\_2EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebool\_2E$

Let  $c\_2Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \quad (44)$$

**Definition 48** We define  $c\_2Enumeral\_2EiZ$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum)^{(ty\_2Ebool\_2Eitself\ A\_27a)} \quad (45)$$

**Definition 49** We define  $c\_2Ewords\_2Eword\_2comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a)$ .

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1k \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1k)\ V0n)) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n) = V1k)))) \quad (46)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ V0n)) \Rightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD\ c\_2Enum\_2E0)\ V0n) = c\_2Enum\_2E0))) \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & ((\forall V0a \in ty\_2Erealax\_2Ereal. \\ & (\forall V1f \in (A\_27a)^{ty\_2Erealax\_2Ereal}).(\forall V2v \in A\_27a. \\ & (\forall V3v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\ & A\_27a)\ (ap\ c\_2Ebinary\_ieee\_2Efloat\ V0a))\ V1f)\ V2v)\ V3v1) = (ap \\ & V1f\ V0a)))))) \wedge ((\forall V4f \in (A\_27a)^{ty\_2Erealax\_2Ereal}).(\forall V5v \in \\ & A\_27a.(\forall V6v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\ & A\_27a)\ c\_2Ebinary\_ieee\_2EInfinity)\ V4f)\ V5v)\ V6v1) = V5v)))) \wedge \\ & (\forall V7f \in (A\_27a)^{ty\_2Erealax\_2Ereal}).(\forall V8v \in A\_27a. \\ & (\forall V9v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\ & A\_27a)\ c\_2Ebinary\_ieee\_2ENaN)\ V7f)\ V8v)\ V9v1) = V9v1)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow & \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ A\_27a\ A\_27b).((p\ (ap \\ & (c\_2Ebinary\_ieee\_2Efloat\_is\_zero\ A\_27a\ A\_27b)\ V0x)) \Leftrightarrow (( \\ & ap\ (c\_2Ebinary\_ieee\_2Efloat\_Exponent\ A\_27a\ A\_27b)\ V0x) = (ap \\ & (c\_2Ewords\_2En2w\ A\_27b)\ c\_2Enum\_2E0)) \wedge ((ap\ (c\_2Ebinary\_ieee\_2Efloat\_Significand \\ & A\_27a\ A\_27b)\ V0x) = (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0)))) \end{aligned} \quad (49)$$

Assume the following.

$$True \quad (50)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2. ((p V0t) \vee (\neg(p V0t)))) \quad (53)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2. (((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (55)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (56)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (57)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (59)$$



Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ & A\_27a. (((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2ET)\ V0t1) \\ & V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ c\_2Ebool\_2EF) \\ & V0t1)\ V1t2) = V1t2)))))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A\_27a. (\forall V3x\_27 \in A\_27a. (\forall V4y \in A\_27a. \\ & (\forall V5y\_27 \in A\_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y\_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND\ A\_27a)\ V1Q)\ V3x\_27) \\ & V5y\_27))))))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V2n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmetic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2A ( \\
& \quad ap c\_2Earithmetic\_2ENUMERAL V6n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum.(\forall V11m \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V10n)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmetic\_2ENUMERAL (ap (ap c\_2Earithmetic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum.((ap \\
& \quad (ap c\_2Earithmetic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum.((ap (ap c\_2Earithmetic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum.(\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmetic\_2EEXP (ap c\_2Earithmetic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V16m)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmetic\_2EEXP V15n) V16m)))))) \wedge ((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad c\_2Earithmetic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL V17n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))) \wedge ((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum.((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL V18n)) = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmetic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmetic\_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum.((c\_2Enum\_2E0 = (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmetic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum.(((ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmetic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum.(\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmetic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmetic\_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum.(\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3E (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmetic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Earithmetic\_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earithmetic\_2EZERO = (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c\_2Earithmetic\_2EBIT1\ V0n) = c\_2Earithmetic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((c\_2Earithmetic\_2EZERO = (ap\ c\_2Earithmetic\_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmetic\_2EBIT2\ V0n) = c\_2Earithmetic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c\_2Earithmetic\_2EBIT1\ V0n) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmetic\_2EBIT2\ V0n) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmetic\_2EBIT1\ V0n) = (ap\ c\_2Earithmetic\_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Earithmetic\_2EBIT2\ V0n) = (ap\ c\_2Earithmetic\_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m)))))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0) \\
& (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a))))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ ( \\
& ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) \\
& (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0m \in ty\_2Enum\_2Enum. ( \\
& \forall V1n \in ty\_2Enum\_2Enum. (((ap\ (c\_2Ewords\_2En2w\ A\_27a)\ V0m) = \\
& (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ V1n)) \Leftrightarrow ((ap\ (ap\ c\_2Earithmetic\_2EMOD \\
& V0m)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_value \\
& A\_27a))) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1n)\ (ap\ (c\_2Ewords\_2Edimword \\
& A\_27a)\ (c\_2Ebool\_2Ethe\_value\ A\_27a))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((ap\ (c\_2Ewords\_2Eword\_2comp \\
& A\_27a)\ (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ (ap\ c\_2Earithmetic\_2ENUMERAL \\
& (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))) = (c\_2Ewords\_2Eword\_T \\
& A\_27a))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0v \in (ty\_2Efc\_2Ecart \\
& 2\ A\_27a). (((ap\ (c\_2Ewords\_2Eword\_2comp\ A\_27a)\ V0v) = (ap\ (c\_2Ewords\_2En2w \\
& A\_27a)\ c\_2Enum\_2E0)) \Leftrightarrow (V0v = (ap\ (c\_2Ewords\_2En2w\ A\_27a)\ c\_2Enum\_2E0))))
\end{aligned} \tag{69}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \\ & \forall V0x \in (ty\_2Ebinary\_ieee\_2Efloat\ A\_27a\ A\_27b).((p\ (ap \\ & (c\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27a\ A\_27b)\ V0x)) \Leftrightarrow ( \\ & (p\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_is\_normal\ A\_27a\ A\_27b)\ V0x)) \vee \\ & ((p\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_is\_subnormal\ A\_27a\ A\_27b) \\ & V0x)) \vee (p\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_is\_zero\ A\_27a\ A\_27b) \\ & V0x)))))) \end{aligned}$$