

thm_2Ebinary_2Eieee_2Efloat_2Eround_2Ebottom (TMRf6sZ7AtjZiJFSFfE2yDCaqeSrVkaPVZZ)

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Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (1)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (2)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c_2Ebool_2Ethe_value\ A.27a \in (ty_2Ebool_2Eitself\ A.27a) \quad (3)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (4)$$

Let $c_2Ebinary_2Eieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27t.nonempty\ A.27t \Rightarrow \forall A.27w.nonempty\ A.27w \Rightarrow c_2Ebinary_2Eieee_2Elargest\ A.27t\ A.27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A.27t\ A.27w))}) \quad (5)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (6)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \quad (7)$$

Definition 1 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal_REP)))$

Let $c_2Erealax_2Etreax_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (8)$$

Let $c_2Erealax_2Etreax_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (9)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (10)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_neg)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (11)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (12)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (13)$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (14)$$

Let $c_2Erealax_2Etreax_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_lt \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (15)$$

Definition 9 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 10 We define c_2Ebool_2EF to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E$

Definition 13 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 14 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2.$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 16 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Efcf_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_2Efcf_2Edimindex A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself A_27a)}) \quad (16)$$

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (18)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_2Enum_2EABS_num$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Definition 19 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 20 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (20)$$

Let $ty_2Efcf_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Efcf_2Ecart A0 A1) \quad (21)$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Ebinary_ieee_2Efloat A0 A1) \quad (22)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efcp_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (23)$$

Let $ty_2Efcp_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcp_2Efinite_image\ A0) \quad (24)$$

Definition 21 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 22 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_2Ebool_2E_3F_21$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ c_2Ebool_2E_2F_50$

Definition 24 We define $c_2Efcp_2Efinite_index$ to be $\lambda A_27a : \iota. (ap\ (c_2Emin_2E_40\ (A_27a\ ^{ty_2Enum_2Enum$

Let $c_2Efcp_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Efcp_2Edest_cart\ A_27a\ A_27b \in ((A_27a\ ^{(ty_2Efcp_2Efinite_image\ A_27b)})^{(ty_2Efcp_2Ecart\ A_27a\ A_27b)}) \quad (25)$$

Definition 25 We define $c_2Efcp_2Efcp_index$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2Efcp_2Ecart\ A_27a$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum\ ^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (26)$$

Definition 26 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2. \lambda V1n \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebo$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum\ ^{(ty_2Enum_2Enum\ ^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (27)$$

Definition 27 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2Efcp_2Ecart\ 2\ A_27a). (ap\ (ap\ c$

Let $c_2Erealax_2Etreax_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (28)$$

Definition 28 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. (ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etreax_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreax_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (29)$$

Definition 29 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 30 We define $c_Ereal_E_2F$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Definition 31 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap (ap c_Earithmic_EBIT1) n)$

Let $c_Erealax_Etrealm_add : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_add \in (((ty_Epair_Eprod ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal) \quad (30)$$

Definition 32 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Let $c_Ewords_EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Ewords_EINT_MAX A_27a \in (ty_Eenum_Eenum)^{(ty_Ebool_Eitself A_27a)} \quad (31)$$

Let $c_Ebinary_ieee_Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_Ebinary_ieee_Efloat_Exponent A_27t A_27w \in ((ty_Efc_Ecart 2 A_27w)^{(ty_Ebinary_ieee_Efloat A_27t A_27w)}) \quad (32)$$

Let $ty_Eone_Eone : \iota$ be given. Assume the following.

$$nonempty ty_Eone_Eone \quad (33)$$

Let $c_Ebinary_ieee_Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty A_27t \Rightarrow \forall A_27w.nonempty A_27w \Rightarrow c_Ebinary_ieee_Efloat_Sign A_27t A_27w \in ((ty_Efc_Ecart 2 ty_Eone_Eone)^{(ty_Ebinary_ieee_Efloat A_27t A_27w)}) \quad (34)$$

Let $c_Earithmic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmic_EDIV \in ((ty_Eenum_Eenum)^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (35)$$

Definition 33 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum.$

Let $c_Earithmic_E_2D : \iota$ be given. Assume the following.

$$c_Earithmic_E_2D \in ((ty_Eenum_Eenum)^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (36)$$

Let $c_Earithmic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmic_EMOD \in ((ty_Eenum_Eenum)^{ty_Eenum_Eenum})^{ty_Eenum_Eenum} \quad (37)$$

Definition 34 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum.$

Definition 35 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 36 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 37 We define c_2Efcf_2EFCF to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 38 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2Efcf_2EFCF$

Definition 39 We define $c_2Ebina_{ry_ieee_2Efloat_to_real}$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina_{ry_ieee_2Efloat_value} : \iota$

Let $ty_2Ebina_{ry_ieee_2Efloat_value} : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebina_{ry_ieee_2Efloat_value} \quad (38)$$

Let $c_2Ebina_{ry_ieee_2Efloat} : \iota$ be given. Assume the following.

$$c_2Ebina_{ry_ieee_2Efloat} \in (ty_2Ebina_{ry_ieee_2Efloat_value}^{ty_2Erealax_2Ereal}) \quad (39)$$

Let $c_2Ebina_{ry_ieee_2ENaN} : \iota$ be given. Assume the following.

$$c_2Ebina_{ry_ieee_2ENaN} \in ty_2Ebina_{ry_ieee_2Efloat_value} \quad (40)$$

Let $c_2Ebina_{ry_ieee_2EInfinity} : \iota$ be given. Assume the following.

$$c_2Ebina_{ry_ieee_2EInfinity} \in ty_2Ebina_{ry_ieee_2Efloat_value} \quad (41)$$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (42)$$

Definition 40 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota.(ap (c_2Ewords_2En2w\ A_27a) (ap (c_2Ew$

Definition 41 We define $c_2Ebina_{ry_ieee_2Efloat_value}$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina_{ry_ieee_2Efloat_value} : \iota$

Let $c_2Ebina_{ry_ieee_2Efloat_value_CASE} : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebina_{ry_ieee_2Efloat_value_CASE}\ A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2Ebina_{ry_ieee_2Efloat_value}} \quad (43)$$

Definition 42 We define $c_2Ebina_{ry_ieee_2Efloat_is_finite}$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebina_{ry_ieee_2Efloat_value} : \iota$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (44)$$

Definition 43 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2E$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (45)$$

Definition 44 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 45 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 46 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Definition 47 We define $c_2Ebinary_ieeee_2Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_2Ebinary_ieeee_2Eis_closest)})$

Definition 48 We define $c_2Ebinary_ieeee_2Eclosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_2Ebinary_ieeee_2Eclosest_such)})$

Definition 49 We define $c_2Ebinary_ieeee_2Eclosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_2Ebinary_ieeee_2Eclosest))$

Let $c_2Ebinary_ieeee_2Efloat_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieeee_2Efloat_top \\ A_27t\ A_27w \in ((ty_2Ebinary_ieeee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (46)$$

Definition 50 We define $c_2Ereal_2Ereal_gt$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieeee_2Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieeee_2Efloat_bottom \\ A_27t\ A_27w \in ((ty_2Ebinary_ieeee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (47)$$

Definition 51 We define c_2Ebool_2ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27a))$

Let $c_2Ebinary_ieeee_2Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieeee_2Efloat_minus_infinity \\ A_27t\ A_27w \in ((ty_2Ebinary_ieeee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (48)$$

Definition 52 We define $c_2Ereal_2Ereal_ge$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Ebinary_ieeee_2Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieeee_2Efloat_plus_infinity \\ A_27t\ A_27w \in ((ty_2Ebinary_ieeee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (49)$$

Let $c_2Ebinary_ieeee_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieeee_2Ethreshold \\ A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (50)$$

Definition 53 We define $c_Ewords_Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap$
 Let $ty_2Ebinary_ieee_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ebinary_ieee_2Erounding \quad (51)$$

Let $c_2Ebinary_ieee_2Erounding2num : \iota$ be given. Assume the following.

$$c_2Ebinary_ieee_2Erounding2num \in (ty_2Enum_2Enum^{ty_2Ebinary_ieee_2Erounding}) \quad (52)$$

Definition 54 We define $c_2Ebinary_ieee_2Erounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_2Ebinary_ieee_2$

Definition 55 We define $c_2Ebinary_ieee_2Eround$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebinary_ie$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \quad (53)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \quad (54)$$

Definition 56 We define $c_2Ebinary_ieee_2Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Definition 57 We define $c_2Ebinary_ieee_2Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_2Ebin$

Definition 58 We define $c_2Ebinary_ieee_2Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebin$

Definition 59 We define $c_2Ebinary_ieee_2Efloat_is_subnormal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2E$

Definition 60 We define $c_2Ebinary_ieee_2Efloat_is_normal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebin$

Definition 61 We define $c_2Ebinary_ieee_2Efloat_is_nan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2Ebinary$

Definition 62 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27$

Definition 63 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap\ (ap\ (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A$

Assume the following.

$$\begin{aligned}
& \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\
& \quad (\neg(p\ (ap\ (c_2Ebinary_ieee_2Efloat_is_zero\ A_27t\ A_27w)\ (ap \\
& \quad (c_2Ebinary_ieee_2Efloat_bottom\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \wedge ((p\ (ap\ (c_2Ebinary_ieee_2Efloat_is_finite \\
& \quad A_27t\ A_27w)\ (ap\ (c_2Ebinary_ieee_2Efloat_bottom\ A_27t\ A_27w) \\
& \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \wedge \\
& \quad ((\neg(p\ (ap\ (c_2Ebinary_ieee_2Efloat_is_nan\ A_27t\ A_27w)\ (ap \\
& \quad (c_2Ebinary_ieee_2Efloat_bottom\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \wedge ((p\ (ap\ (c_2Ebinary_ieee_2Efloat_is_normal \\
& \quad A_27t\ A_27w)\ (ap\ (c_2Ebinary_ieee_2Efloat_bottom\ A_27t\ A_27w) \\
& \quad (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \Leftrightarrow \\
& \quad (\neg((ap\ (c_2Efc_2Edimindex\ A_27w)\ (c_2Ebool_2Ethe_value\ A_27w)) = \\
& \quad (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad (((p\ (ap\ (c_2Ebinary_ieee_2Efloat_is_subnormal\ A_27t\ A_27w) \\
& \quad (ap\ (c_2Ebinary_ieee_2Efloat_bottom\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \Leftrightarrow ((ap\ (c_2Efc_2Edimindex \\
& \quad A_27w)\ (c_2Ebool_2Ethe_value\ A_27w)) = (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \wedge (\neg(p\ (\\
& \quad ap\ (c_2Ebinary_ieee_2Efloat_is_infinite\ A_27t\ A_27w)\ (ap \\
& \quad (c_2Ebinary_ieee_2Efloat_bottom\ A_27t\ A_27w)\ (c_2Ebool_2Ethe_value \\
& \quad (ty_2Epair_2Eprod\ A_27t\ A_27w)))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$True \tag{56}$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{57}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \tag{58}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}). (\forall V1x \in A_27a. ((ap\ (ap\ (c_2Ebool_2ELET \\
& \quad A_27a\ A_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& \quad True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\
& \quad (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (62)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (63)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2. (\forall V1x_27 \in 2. (\forall V2y \in 2. (\forall V3y_27 \in 2. \\ & 2. (((p\ V0x) \Leftrightarrow (p\ V1x_27)) \wedge ((p\ V1x_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y_27)))))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x_27) \Rightarrow (p\ V3y_27)))))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. \\ & (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. \\ & (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge ((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge \\ & ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a) \\ & V0P)\ V2x)\ V4y) = (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27 \\ & V5y_27)))))) \end{aligned} \quad (65)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in \\ & A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap \\ & (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \end{aligned} \quad (66)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap\ (c_2Ecombin_2EI\ A_27a)\ V0x) = V0x)) \quad (67)$$

Theorem 1

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\ & \forall V0mode \in ty_2Ebinary_ieee_2Erounding. (\forall V1toneg \in \\ & 2. (\forall V2r \in ty_2Erealax_2Ereal. (((ap\ (ap\ (c_2Ebinary_ieee_2ERound \\ & A_27t\ A_27w)\ V0mode)\ V2r) = (ap\ (c_2Ebinary_ieee_2Efloat_bottom \\ & A_27t\ A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t \\ & A_27w)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebinary_ieee_2Efloat_round\ A_27t \\ & A_27w)\ V0mode)\ V1toneg)\ V2r) = (ap\ (c_2Ebinary_ieee_2Efloat_bottom \\ & A_27t\ A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t \\ & A_27w)))))) \end{aligned}$$