thm_2Ebinary__ieee_2Efloat__round__minus__infinity (TMdBLCTBFyi-ayyEn6Vc7Nz3PYbKUWokkCoy)

October 26, 2020

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{1}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{2}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{3}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $c\_2Ebinary\_\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Elargest\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \tag{5}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{7}$$

Definition 1 *We define* $\mathsf{c\_2Emin\_2E\_40}$ *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* $\mathsf{c\_2Emin\_2E\_3D}$ *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* $\mathsf{c\_2Ebool\_2ET}$ *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* $\mathsf{c\_2Ebool\_2E\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_REP}$ *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{8}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{9}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \tag{10}$$

Definition 6 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_ABS}$ *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 7 *We define* $\mathsf{c\_2Erealax\_2Ereal\_\_neg}$ *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_}$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{11}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{12}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{13}$$

Definition 8 *We define* $\mathsf{c\_2Enum\_2E0}$ *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{15}$$

Definition 9 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 12 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 13 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECON}$

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{16}$$

Definition 17 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{17}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{18}$$

Definition 18 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 19 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 20 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{20}$$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{21}$$

Let $ty\_2Ebinary\_\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A0\ A1) \tag{22}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{23}$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{24}$$

Definition 21 *We define* $\mathsf{c\_2Ebool\_2E\_3F}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 22 *We define* $\mathsf{c\_2Eprim\_\_rec\_2E\_3C}$ *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 23 *We define* $\mathsf{c\_2Ebool\_2E\_3F\_21}$ *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}$

Definition 24 *We define* $\mathsf{c\_2Efcp\_2Efinite\_\_index}$ *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (A\_27a^{ty\_2Enum\_2Enu}$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{25}$$

Definition 25 *We define* $\mathsf{c\_2Efcp\_2Efcp\_\_index}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{26}$$

Definition 26 *We define* $\mathsf{c\_2Ebit\_2ESBIT}$ *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (\mathsf{c\_2Eboo}$

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \tag{27}$$

Definition 27 *We define* $\mathsf{c\_2Ewords\_2Ew2n}$ *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{28}$$

Definition 28 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (29)$$

Definition 29 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 30 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 31 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (30)$$

Definition 32 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Ewords\_2EINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_2} \quad (31)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (32)$$

Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (33)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (34)$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (35)$$

Definition 33 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{36}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{37}$$

Definition 34 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 35 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 36 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 37 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 38 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 39 *We define* c_2Ebinary__ieee_2Efloat__to__real *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let $ty\_2Ebinary\_\_ieee\_2Efloat\_\_value : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{38}$$

Let $c\_2Ebinary\_\_ieee\_2EFloat : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EFloat \in (ty\_2Ebinary\_\_ieee\_2Efloat\_\_value^{ty\_2Erealax\_2Ereal}) \tag{39}$$

Let $c\_2Ebinary\_\_ieee\_2ENaN : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2ENaN \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{40}$$

Let $c\_2Ebinary\_\_ieee\_2EInfinity : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EInfinity \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{41}$$

Let $c\_2Ewords\_2EUINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{42}$$

Definition 40 *We define* c_2Ewords_2Eword__T *to be* $\lambda A\_27a : \iota.(ap$ (c_2Ewords_2En2w A_27a) $(ap\ (c\_2Ew$

Definition 41 *We define* c_2Ebinary__ieee_2Efloat__value *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE\ A\_27a \in ((((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_\_ieee\_2Efloat\_\_value}) \tag{43}$$

Definition 42 *We define* c_2Ebinary__ieee_2Efloat__is__finite *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (44)$$

Definition 43 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (45)$$

Definition 44 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 45 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Definition 46 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 47 *We define* c_2Ebinary__ieee_2Eis__closest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebinary\_\_ie}$

Definition 48 *We define* c_2Ebinary__ieee_2Eclosest__such *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebinary}$

Definition 49 *We define* c_2Ebinary__ieee_2Eclosest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap$ (c_2Ebinary__ieee_2Eclos

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_top\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (46)$$

Definition 50 *We define* c_2Ereal_2Ereal__gt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (47)$$

Definition 51 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (48)$$

Definition 52 *We define* c_2Ereal_2Ereal__ge *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{49}$$

Let $c\_2Ebinary\_\_ieee\_2Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Ethreshold\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \tag{50}$$

Definition 53 *We define* c_2Ewords_2Eword__lsb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Let $ty\_2Ebinary\_\_ieee\_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Erounding \tag{51}$$

Let $c\_2Ebinary\_\_ieee\_2Erounding2num : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_\_ieee\_2Erounding}) \tag{52}$$

Definition 54 *We define* c_2Ebinary__ieee_2Erounding__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_\_ieee\_2$

Definition 55 *We define* c_2Ebinary__ieee_2Eround *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_\_ie$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{53}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{54}$$

Definition 56 *We define* c_2Ebinary__ieee_2Efloat__is__zero *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Definition 57 *We define* c_2Ebinary__ieee_2Efloat__round *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebina$

Definition 58 *We define* c_2Ebinary__ieee_2Efloat__is__infinite *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

Definition 59 *We define* c_2Ebinary__ieee_2Efloat__is__subnormal *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2E$

Definition 60 *We define* c_2Ebinary__ieee_2Efloat__is__normal *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

Definition 61 *We define* c_2Ebinary__ieee_2Efloat__is__nan *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Definition 62 *We define* c_2Ebinary__ieee_2Eis__integral *to be* $\lambda V0r \in ty\_2Erealax\_2Ereal.(ap$ (c_2Ebool_2

Definition 63 *We define* c_2Ebinary__ieee_2Efloat__is__integral *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebi$

Definition 64 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 65 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_zero}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_zero}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_finite}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_finite}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_integral}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_integral}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_nan}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_nan}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_normal}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_normal}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_subnormal}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((\neg(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_subnormal}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))\wedge((p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_infinite}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))))\wedge(p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_infinite}\ A\_27t\ A\_27w)\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))))))))))))))) \tag{55}$$

Assume the following.

$$True \tag{56}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{57}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \tag{59}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{60}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{62}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{63}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{64}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in 2.(\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a.(\forall V5y\_27 \in A\_27a.((((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x\_27)) \wedge ((\neg (p\ V1Q)) \Rightarrow (V4y = V5y\_27)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{65}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A\_27a.(\forall V3t2 \in A\_27a.((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V2t1)\ V3t2) = V3t2)))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{67}$$

Theorem 1

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow (\forall V0mode \in ty\_2Ebinary\_\_ieee\_2Erounding.(\forall V1toneg \in 2.(\forall V2r \in ty\_2Erealax\_2Ereal.((((ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Eround}\ A\_27t\ A\_27w)\ V0mode)\ V2r) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_round}\ A\_27t\ A\_27w)\ V0mode)\ V1toneg)\ V2r) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))))))))$$