thm_2Ebinary__ieee_2Efloat__round__non__zero (TMdGA3kJknzMg7Y7URZ9gKP3JFNsZz2wQqC)

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Let $ty\_2Eone\_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \tag{1}$$

Let $ty\_2Efcp\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Efcp\_2Ecart\ A0\ A1) \tag{2}$$

Let $ty\_2Ebinary\_\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A0\ A1) \tag{3}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{4}$$

Let $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2EARB\ A\_27a \in A\_27a \tag{5}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27u\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27u\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{(\tag{6}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow \forall A\_27x.nonempty\ A\_27x \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27 \ldots A\_27w\ A\_27x \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27x)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc \tag{7}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_ A\_27t\ A\_27w \in (((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)})^{((ty\_2Efc} \tag{8}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{9}$$

Let $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ebool\_2Eitself\ A0) \tag{10}$$

Let $c\_2Ebool\_2Ethe\_\_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebool\_2Ethe\_\_value\ A\_27a \in (ty\_2Ebool\_2Eitself\ A\_27a) \tag{11}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{12}$$

Let $c\_2Ebinary\_\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Elargest\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \tag{13}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{14}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{15}$$

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A_27}$

Definition 5 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ ($c_2Emin_2E_40$\ (ty$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (16)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (17)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \quad (18)$$

Definition 6 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 7 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (19)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (20)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (21)$$

Definition 8 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (22)$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (23)$$

Definition 9 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2$

Definition 10 *We define* c_2Ebool_2EF *to be* $(ap$ (c_2Ebool_2E_21 2) $(\lambda V0t \in 2.V0t))$.

Definition 11 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 12 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 13 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 14 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 15 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 16 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Efcp\_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Efcp\_2Edimindex\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{24}$$

Definition 17 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{25}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{26}$$

Definition 18 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{27}$$

Definition 19 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 20 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{28}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{29}$$

Let $ty\_2Efcp\_2Efinite\_\_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Efcp\_2Efinite\_\_image\ A0) \tag{30}$$

Definition 21 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 22 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 23 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Definition 24 *We define* c_2Efcp_2Efinite__index *to be* $\lambda A\_27a : \iota.(ap$ (c_2Emin_2E_40 $(A\_27a^{ty\_2Enum\_2Enu$

Let $c\_2Efcp\_2Edest\_\_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Efcp\_2Edest\_\_cart\ A\_27a\ A\_27b \in ((A\_27a^{(ty\_2Efcp\_2Efinite\_\_image\ A\_27b)})^{(ty\_2Efcp\_2Ecart\ A\_27a\ A\_27b)}) \tag{31}$$

Definition 25 *We define* c_2Efcp_2Efcp__index *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcp\_2Ecart\ A\_27$

Let $c\_2Earithmetic\_2EEXP : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{32}$$

Definition 26 *We define* c_2Ebit_2ESBIT *to be* $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap$ (c_2Eboo

Let $c\_2Esum\_\_num\_2ESUM : \iota$ be given. Assume the following.

$$c\_2Esum\_\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \tag{33}$$

Definition 27 *We define* c_2Ewords_2Ew2n *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap\ (ap\ c$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{34}$$

Definition 28 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{35}$$

Definition 29 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 30 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 31 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{36}$$

Definition 32 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Ewords\_2EINT\_\_MAX : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Ewords\_2EINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_2 \tag{37}$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27w.nonempty\ A\_27w{\Rightarrow}c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \tag{38}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{39}$$

Definition 33 *We define* c_2Ebit_2EDIV__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{40}$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{41}$$

Definition 34 *We define* c_2Ebit_2EMOD__2EXP *to be* $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2E$

Definition 35 *We define* c_2Ebit_2EBITS *to be* $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

Definition 36 *We define* c_2Ebit_2EBIT *to be* $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

Definition 37 *We define* c_2Efcp_2EFCP *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

Definition 38 *We define* c_2Ewords_2En2w *to be* $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap$ (c_2Efcp_2EFC

Definition 39 *We define* c_2Ebinary__ieee_2Efloat__to__real *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Let $ty\_2Ebinary\_\_ieee\_2Efloat\_\_value : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{42}$$

Let $c\_2Ebinary\_\_ieee\_2EFloat : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EFloat \in (ty\_2Ebinary\_\_ieee\_2Efloat\_\_value^{ty\_2Erealax\_2Ereal}) \tag{43}$$

Let $c\_2Ebinary\_\_ieee\_2ENaN : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2ENaN \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \tag{44}$$

Let $c\_2Ebinary\_\_ieee\_2EInfinity : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2EInfinity \in ty\_2Ebinary\_\_ieee\_2Efloat\_\_value \quad (45)$$

Let $c\_2Ewords\_2EUINT\_\_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (46)$$

Definition 40 *We define* c_2Ewords_2Eword__T *to be* $\lambda A\_27a : \iota.(ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ (ap\ (c\_2Ew$

Definition 41 *We define* c_2Ebinary__ieee_2Efloat__value *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_value\_\_CASE\ A\_27a \in ((((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_2Erealax\_2Ereal})})^{ty\_2Ebinary\_\_ieee\_2Efloat\_\_value}) \quad (47)$$

Definition 42 *We define* c_2Ebinary__ieee_2Efloat__is__finite *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebina$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (48)$$

Definition 43 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \quad (49)$$

Definition 44 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 45 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Definition 46 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Definition 47 *We define* c_2Ebinary__ieee_2Eis__closest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebinary\_\_ie}$

Definition 48 *We define* c_2Ebinary__ieee_2Eclosest__such *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebinary}$

Definition 49 *We define* c_2Ebinary__ieee_2Eclosest *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Eclos}$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_top : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_top\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (50)$$

Definition 50 *We define* c_2Ereal_2Ereal__gt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_bottom\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (51)$$

Definition 51 *We define* c_2Ebool_2ELET *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b^{A\_27a}).(\lambda V1x \in A\_27$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (52)$$

Definition 52 *We define* c_2Ereal_2Ereal__ge *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_infinity\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (53)$$

Let $c\_2Ebinary\_\_ieee\_2Ethreshold : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Ethreshold\ A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \quad (54)$$

Definition 53 *We define* c_2Ewords_2Eword__lsb *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(ap$

Let $ty\_2Ebinary\_\_ieee\_2Erounding : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_\_ieee\_2Erounding \quad (55)$$

Let $c\_2Ebinary\_\_ieee\_2Erounding2num : \iota$ be given. Assume the following.

$$c\_2Ebinary\_\_ieee\_2Erounding2num \in (ty\_2Enum\_2Enum^{ty\_2Ebinary\_\_ieee\_2Erounding}) \quad (56)$$

Definition 54 *We define* c_2Ebinary__ieee_2Erounding__CASE *to be* $\lambda A\_27a : \iota.\lambda V0x \in ty\_2Ebinary\_\_ieee\_2$

Definition 55 *We define* c_2Ebinary__ieee_2Eround *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary\_\_ie$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero : \iota\Rightarrow\iota\Rightarrow\iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t\Rightarrow\forall A\_27w.nonempty\ A\_27w\Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_plus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \quad (57)$$

Let $c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_\_ieee\_2Efloat\_\_minus\_\_zero\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)} \tag{58}$$

Definition 56 *We define* c_2Ebinary__ieee_2Efloat__is__zero *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinar$

Definition 57 *We define* c_2Ebinary__ieee_2Efloat__round *to be* $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebina$

Definition 58 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 59 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 60 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap$ (c_2Ecombin_2ES $A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Ewords\_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2Edimword\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \tag{59}$$

Assume the following.

$$
\begin{array}{c}
\forall A\_27t.nonempty\ A\_27t{\Rightarrow}\forall A\_27u.nonempty\ A\_27u{\Rightarrow}\forall A\_27w.\\
nonempty\ A\_27w{\Rightarrow}\forall A\_27x.nonempty\ A\_27x{\Rightarrow}((\forall V0f0 \in\\
((ty\_2Efcp\_2Ecart\ 2\ A\_27x)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27w)}).(\forall V1f \in\\
(ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\\
A\_27t\ A\_27x)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\\
A\_27t\ A\_27w\ A\_27x)\ V0f0)\ V1f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\\
A\_27t\ A\_27w)\ V1f))))\wedge((\forall V2f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27u)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)}).\\
(\forall V3f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27u\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fu\\
A\_27t\ A\_27u\ A\_27w)\ V2f0)\ V3f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\\
A\_27t\ A\_27w)\ V3f))))\wedge((\forall V4f0 \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eo}\\
(\forall V5f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\\
A\_27t\ A\_27w)\ V4f0)\ V5f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\\
A\_27t\ A\_27w)\ V5f))))\wedge((\forall V6f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27u)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)}).\\
(\forall V7f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27u\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_\\
A\_27t\ A\_27u\ A\_27w)\ V6f0)\ V7f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\\
A\_27t\ A\_27w)\ V7f))))\wedge((\forall V8f0 \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eo}\\
(\forall V9f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w)\ (ap\ (ap\ (\\
c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27t\ A\_27w)\ V8f0)\ V9f)) =\\
(ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27w)\ V9f))))\wedge\\
((\forall V10f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27x)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27w)}).\\
(\forall V11f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\ A\_27x)\ (ap\ (ap\\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t\ A\_27w\ A\_27x)\\
V10f0)\ V11f)) = (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27t\\
A\_27w)\ V11f))))\wedge((\forall V12f0 \in ((ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone}\\
(\forall V13f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\ A\_27t\ A\_27w)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\\
A\_27t\ A\_27w)\ V12f0)\ V13f)) = (ap\ V12f0\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\\
A\_27t\ A\_27w)\ V13f)))))\wedge((\forall V14f0 \in ((ty\_2Efcp\_2Ecart\ 2\\
A\_27x)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27w)}).(\forall V15f \in (ty\_2Ebinary\_\_ieee\_2Efloat\\
A\_27t\ A\_27w).((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27t\\
A\_27x)\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27t\\
A\_27w\ A\_27x)\ V14f0)\ V15f)) = (ap\ V14f0\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\\
A\_27t\ A\_27w)\ V15f)))))\wedge(\forall V16f0 \in ((ty\_2Efcp\_2Ecart\ 2\ A\_27u)^{(ty\_2Efcp\_2Ecart\ 2\ A\_27t)}).\\
(\forall V17f \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w).((ap\\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27u\ A\_27w)\ (ap\ (ap\\
(c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27u\ A\_27w)\\
V16f0)\ V17f)) = (ap\ V16f0\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\\
A\_27t\ A\_27w)\ V17f)))))))))))))
\end{array}
\tag{60}
$$

Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27u.nonempty\ A\_27u \Rightarrow \forall A\_27w.\ nonempty\ A\_27w \Rightarrow \forall A\_27x.nonempty\ A\_27x \Rightarrow (\forall V0c11 \in (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V1c01 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27x).(\forall V2c1 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27u).(\forall V3c12 \in (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V4c02 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27x).(\forall V5c2 \in (ty\_2Efcp\_2Ecart\ 2\ A\_27u).(((ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27u\ A\_27x)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone))\ V0c11))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27u\ A\_27w\ A\_27x)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27x)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w))\ V1c01))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27u\ A\_27w)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27u)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27t))\ V2c1))\ (c\_2Ebool\_2EARB\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w))))) = (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27u\ A\_27x)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone))\ V3c12))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27u\ A\_27w\ A\_27x)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27x)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27w))\ V4c02))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27t\ A\_27u\ A\_27w)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27u)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27t))\ V5c2))\ (c\_2Ebool\_2EARB\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27t\ A\_27w))))) \Leftrightarrow ((V0c11 = V3c12) \land ((V1c01 = V4c02) \land (V2c1 = V5c2)))))))))) \tag{61}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27a\ A\_27b).((p\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_is\_\_zero}\ A\_27a\ A\_27b)\ V0x)) \Leftrightarrow (((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\ A\_27a\ A\_27b)\ V0x) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27b)\ \mathsf{c\_2Enum\_2E0})) \land ((ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\ A\_27a\ A\_27b)\ V0x) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))))) \tag{62}$$

Assume the following.

$$True \tag{63}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{64}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{65}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg (p\ V0t)))) \tag{66}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1x \in A\_27a.((ap\ (ap\ (\mathsf{c\_2Ebool\_2ELET}\ A\_27a\ A\_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x)))) \tag{67}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{68}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{69}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{70}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{71}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{72}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{73}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{74}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{75}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1)\Rightarrow ((p\ V1t2)\Rightarrow(p\ V2t3)))\Leftrightarrow(((p\ V0t1)\wedge(p\ V1t2))\Rightarrow(p\ V2t3)))))) \tag{76}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x)\Leftrightarrow(p\ V1x\_27))\wedge((p\ V1x\_27)\Rightarrow((p\ V2y)\Leftrightarrow(p\ V3y\_27))))\Rightarrow (((p\ V0x)\Rightarrow(p\ V2y))\Leftrightarrow((p\ V1x\_27)\Rightarrow(p\ V3y\_27)))))))) \tag{77}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0P \in 2.(\forall V1Q \in 2. (\forall V2x \in A\_27a.(\forall V3x\_27 \in A\_27a.(\forall V4y \in A\_27a. (\forall V5y\_27 \in A\_27a.((((p\ V0P)\Leftrightarrow(p\ V1Q))\wedge(((p\ V1Q)\Rightarrow(V2x = V3x\_27))\wedge ((\neg(p\ V1Q))\Rightarrow(V4y = V5y\_27))))\Rightarrow((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ V1Q)\ V3x\_27)\ V5y\_27))))))))) \tag{78}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow\forall A\_27b.nonempty\ A\_27b\Rightarrow(\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ A\_27a\ A\_27b)\ V0x)\ V1y) = V0x))) \tag{79}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{80}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a\Rightarrow(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(((ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V0m) = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ V1n))\Leftrightarrow((ap\ (ap\ c\_2Earithmetic\_2EMOD\ V0m)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))) = (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1n)\ (ap\ (c\_2Ewords\_2Edimword\ A\_27a)\ (c\_2Ebool\_2Ethe\_\_value\ A\_27a))))))) \tag{81}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0mode \in ty\_2Ebinary\_\_ieee\_2Erounding.(\forall V1toneg \in 2.(\forall V2r \in ty\_2Erealax\_2Ereal.(\forall V3s \in (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone).(\forall V4e \in (ty\_2Efcp\_2Ecart\ 2\ A\_27a).(\forall V5f \in (ty\_2Efcp\_2Ecart\ 2\ A\_27b).((((ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Eround}\ A\_27b\ A\_27a)\ V0mode)\ V2r) = (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone))\ V3s))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27b\ A\_27a\ A\_27a)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ V4e))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27b\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27b)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27b))\ V5f))\ (c\_2Ebool\_2EARB\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27b\ A\_27a))))))) \land ((\neg(V4e = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \lor (\neg(V5f = (ap\ (\mathsf{c\_2Ewords\_2En2w}\ A\_27b)\ \mathsf{c\_2Enum\_2E0}))))) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebinary\_\_ieee\_2Efloat\_\_round}\ A\_27b\ A\_27a)\ V0mode)\ V1toneg)\ V2r) = (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Sign\_\_fupd\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone)\ (ty\_2Efcp\_2Ecart\ 2\ ty\_2Eone\_2Eone))\ V3s))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Exponent\_\_fupd\ A\_27b\ A\_27a\ A\_27a)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27a))\ V4e))\ (ap\ (ap\ (c\_2Ebinary\_\_ieee\_2Efloat\_\_Significand\_\_fupd\ A\_27b\ A\_27b\ A\_27a)\ (ap\ (\mathsf{c\_2Ecombin\_2EK}\ (ty\_2Efcp\_2Ecart\ 2\ A\_27b)\ (ty\_2Efcp\_2Ecart\ 2\ A\_27b))\ V5f))\ (c\_2Ebool\_2EARB\ (ty\_2Ebinary\_\_ieee\_2Efloat\ A\_27b\ A\_27a)))))))))))))$$