

thm_2Ebinary_ieee_2Efloat_round_roundTowardNegative_minus_ (TMcVF25Q8a97fmNEpGXzoayRTjJt5QyLMJ5)

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Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \quad (1)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (2)$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (3)$$

Let $c_2Ebinary_ieee_2Efloat_plus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_plus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \quad (4)$$

Let $c_2Ebinary_ieee_2Efloat_minus_zero : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_minus_zero\ A_27t\ A_27w \in ((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \quad (5)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (6)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (7)$$

Let $c_2Ebinary_ieee_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Elargest\ A_27t\ A_27w \in (ty_2Erealax_2Ereal^{(ty_2Ebool_2Eitself\ (ty_2Epair_2Eprod\ A_27t\ A_27w))}) \quad (8)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (9)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (10)$$

Definition 1 We define c_2Emin_2E40 to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p\ x)$ of type $\iota \Rightarrow \iota$).

Definition 2 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define c_2Ebool_2E2T to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A_27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x)))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal)))$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (11)$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (12)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (13)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_neg)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (14)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (15)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (16)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (17)$$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (18)$$

Definition 9 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 10 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 11 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 12 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E7E))$

Definition 13 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 14 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2.V2t))))$

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.V2t2)))$

Definition 16 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ x))\ x))$

Let $c_2Efcfcp_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efcfcp_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitsself\ A_27a)}) \quad (19)$$

Definition 17 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (20)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (21)$$

Definition 18 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ m)$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (22)$$

Definition 19 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2E2B\ n))$

Definition 20 We define `c_2Earithmetic_2ENUMERAL` to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let `c_2Ereal_2Epow` : ι be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (23)$$

Let `ty_2EfcP_2Ecart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2EfcP_2Ecart\ A0\ A1) \quad (24)$$

Let `c_2Ebinary_2ieee_2Efloat_2Significand` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2ieee_2Efloat_2Significand\ A_27t\ A_27w \in ((ty_2EfcP_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_2ieee_2Efloat\ A_27t\ A_27w)}) \quad (25)$$

Let `ty_2EfcP_2Efinite_image` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2EfcP_2Efinite_image\ A0) \quad (26)$$

Definition 21 We define `c_2Ebool_2E_3F` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 22 We define `c_2Eprim_rec_2E_3C` to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define `c_2Ebool_2E_3F_21` to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ (ap\ c_2Ebool_2E_2F_5C$

Definition 24 We define `c_2EfcP_2Efinite_index` to be $\lambda A_27a : \iota. (ap\ (c_2Emin_2E_40\ (A_27a^{ty_2Enum_2Enum}$

Let `c_2EfcP_2Edest_cart` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EfcP_2Edest_cart\ A_27a\ A_27b \in ((A_27a^{(ty_2EfcP_2Efinite_image\ A_27b)})^{(ty_2EfcP_2Ecart\ A_27a\ A_27b)}) \quad (27)$$

Definition 25 We define `c_2EfcP_2EfcP_index` to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in (ty_2EfcP_2Ecart\ A_27a$

Let `c_2Earithmetic_2EEXP` : ι be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (28)$$

Definition 26 We define `c_2Ebit_2ESBIT` to be $\lambda V0b \in 2. \lambda V1n \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebo$

Let `c_2Esum_num_2ESUM` : ι be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (29)$$

Definition 27 We define `c_2Ewords_2Ew2n` to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a). (ap\ (ap\ c$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_inv \in & ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (30)$$

Definition 28 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_mul \in & (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (31)$$

Definition 29 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 30 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 31 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic_2EBIT1$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_add \in & (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (32)$$

Definition 32 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (33)$$

Let $c_2Ebinary_2Eieee_2Efloat_2EExponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2Eieee_2Efloat_2EExponent \\ A_27t\ A_27w \in & ((ty_2Efc_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_2Eieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (34)$$

Let $ty_2Eone_2Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eone_2Eone \quad (35)$$

Let $c_2Ebinary_2Eieee_2Efloat_2ESign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_2Eieee_2Efloat_2ESign \\ A_27t\ A_27w \in & ((ty_2Efc_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2Ebinary_2Eieee_2Efloat\ A_27t\ A_27w)}) \end{aligned} \quad (36)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (37)$$

Definition 33 We define $c_2Ebit_2EDIV_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.
Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (38)$$

Let $c_2Earithmetic_2EMOD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EMOD \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (39)$$

Definition 34 We define $c_2Ebit_2EMOD_2EXP$ to be $\lambda V0x \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$.

Definition 35 We define c_2Ebit_2EBITS to be $\lambda V0h \in ty_2Enum_2Enum.\lambda V1l \in ty_2Enum_2Enum.\lambda V$

Definition 36 We define c_2Ebit_2EBIT to be $\lambda V0b \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(ap$

Definition 37 We define c_2EfcP_2EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_2Enum_2Enum}).(ap$

Definition 38 We define $c_2Ewords_2En2w$ to be $\lambda A_27a : \iota.\lambda V0n \in ty_2Enum_2Enum.(ap (c_2EfcP_2EFCP$

Definition 39 We define $c_2EbinaRy_2ieee_2Efloat_2to_2real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2EbinaRy$

Let $ty_2EbinaRy_2ieee_2Efloat_2value : \iota$ be given. Assume the following.

$$nonempty\ ty_2EbinaRy_2ieee_2Efloat_2value \quad (40)$$

Let $c_2EbinaRy_2ieee_2Efloat : \iota$ be given. Assume the following.

$$c_2EbinaRy_2ieee_2Efloat \in (ty_2EbinaRy_2ieee_2Efloat_2value^{ty_2Erealax_2Ereal}) \quad (41)$$

Let $c_2EbinaRy_2ieee_2ENaN : \iota$ be given. Assume the following.

$$c_2EbinaRy_2ieee_2ENaN \in ty_2EbinaRy_2ieee_2Efloat_2value \quad (42)$$

Let $c_2EbinaRy_2ieee_2EInfinity : \iota$ be given. Assume the following.

$$c_2EbinaRy_2ieee_2EInfinity \in ty_2EbinaRy_2ieee_2Efloat_2value \quad (43)$$

Let $c_2Ewords_2EUINT_2MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EUINT_2MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (44)$$

Definition 40 We define $c_2Ewords_2Eword_2T$ to be $\lambda A_27a : \iota.(ap (c_2Ewords_2En2w\ A_27a) (ap (c_2Ew$

Definition 41 We define $c_2EbinaRy_2ieee_2Efloat_2value$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_2EbinaRy$

Let $c_2EbinaRy_2ieee_2Efloat_2value_2CASE : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in (((A_27a^{A_27a})^{A_27a})^{(A_27a^{ty_2Erealax_2Ereal})})^{ty_2EbinaRy_2ieee_2Efloat_2value} \quad (45)$$

Definition 42 We define $c_Ebinary_ieee_Efloat_is_finite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebina$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod \\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (46)$$

Definition 43 We define $c_Epair_E_EC$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2$

Let $c_Epred_set_EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epred_set_EGSPEC \\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_Epair_Eprod\ A_27a\ 2)^{A_27b}}) \end{aligned} \quad (47)$$

Definition 44 We define $c_Ecombin_EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 45 We define $c_Ereal_Ereal_sub$ to be $\lambda V0x \in ty_Erealx_Ereal.\lambda V1y \in ty_Erealx_Ereal$

Definition 46 We define c_Ebool_EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).) (ap\ V1f\ V0x))$

Definition 47 We define $c_Ebinary_ieee_Eis_closest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s \in (2^{(ty_Ebina$

Definition 48 We define $c_Ebinary_ieee_Eclosest_such$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0p \in (2^{(ty_Ebina$

Definition 49 We define $c_Ebinary_ieee_Eclosest$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(ap\ (c_Ebinary_ieee_Eclose$

Let c_Ebina

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebina_ieee_Efloat_top \\ A_27t\ A_27w \in ((ty_Ebina_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (48)$$

Definition 50 We define $c_Ereal_Ereal_gt$ to be $\lambda V0x \in ty_Erealx_Ereal.\lambda V1y \in ty_Erealx_Ereal$

Let $c_Ebina_ieee_Efloat_bottom : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebina_ieee_Efloat_bottom \\ A_27t\ A_27w \in ((ty_Ebina_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (49)$$

Definition 51 We define c_Ebool_ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).)(\lambda V1x \in A_27$

Let $c_Ebina_ieee_Efloat_minus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebina_ieee_Efloat_minus_infinity \\ A_27t\ A_27w \in ((ty_Ebina_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \end{aligned} \quad (50)$$

Definition 52 We define $c_Ereal_Ereal_ge$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal$

Let $c_Ebinary_ieee_Efloat_plus_infinity : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Efloat_plus_infinity\ A_27t\ A_27w \in ((ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (51)$$

Let $c_Ebinary_ieee_Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee_Ethreshold\ A_27t\ A_27w \in (ty_Erealax_Ereal^{(ty_Ebool_Eitself\ (ty_Epair_Eprod\ A_27t\ A_27w))}) \quad (52)$$

Definition 53 We define $c_Ewords_Eword_lsb$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efcpcart\ 2\ A_27a).(ap$

Let $ty_Ebinary_ieee_ERounding : \iota$ be given. Assume the following.

$$nonempty\ ty_Ebinary_ieee_ERounding \quad (53)$$

Let $c_Ebinary_ieee_ERounding2num : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_ERounding2num \in (ty_Eenum_enum^{ty_Ebinary_ieee_ERounding}) \quad (54)$$

Definition 54 We define $c_Ebinary_ieee_ERounding_CASE$ to be $\lambda A_27a : \iota.\lambda V0x \in ty_Ebinary_ieee_E$

Definition 55 We define $c_Ebinary_ieee_ERound$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_Ebinary_ie$

Definition 56 We define $c_Ebinary_ieee_Efloat_is_zero$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebinar$

Definition 57 We define $c_Ebinary_ieee_Efloat_round$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0mode \in ty_Ebin$

Definition 58 We define $c_Ebinary_ieee_Efloat_is_infinite$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebin$

Definition 59 We define $c_Ebinary_ieee_Efloat_is_subnormal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_E$

Definition 60 We define $c_Ebinary_ieee_Efloat_is_normal$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebin$

Definition 61 We define $c_Ebinary_ieee_Efloat_is_nan$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebinar$

Definition 62 We define $c_Ebinary_ieee_Eis_integral$ to be $\lambda V0r \in ty_Erealax_Ereal.(ap\ (c_Ebool_2$

Definition 63 We define $c_Ebinary_ieee_Efloat_is_integral$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebin$

Let $c_Ebinary_ieee_ERoundTowardNegative : \iota$ be given. Assume the following.

$$c_Ebinary_ieee_ERoundTowardNegative \in ty_Ebinary_ieee_ERounding \quad (55)$$

Definition 64 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 65 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES A_27a (A_27a^{A_27a})) A_27a))$

Assume the following.

$$\begin{aligned}
& \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow (\\
& \quad \forall V0y \in (ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w).(\forall V1x \in \\
& \quad ty_2Erealax_2Ereal.((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V1x)\ (ap \\
& \quad c_2Erealax_2Ereal_neg\ (ap\ (c_2Ebinary_ieee_2Elargest\ A_27t \\
& \quad A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \Rightarrow \\
& ((ap\ (ap\ (c_2Ebinary_ieee_2Eround\ A_27t\ A_27w)\ c_2Ebinary_ieee_2EroundTowardNegative) \\
& \quad V1x) = (ap\ (c_2Ebinary_ieee_2Efloat_minus_infinity\ A_27t \\
& \quad A_27w)\ (c_2Ebool_2Ethe_value\ (ty_2Epair_2Eprod\ A_27t\ A_27w)))))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$True \quad (58)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\
& \quad V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \\
& \hspace{15em} (59)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (60)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap\ (ap\ (c_2Ebool_2ELET \\
& \quad A_27a\ A_27b)\ V0f)\ V1x) = (ap\ V0f\ V1x))) \\
& \hspace{15em} (61)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\
& \quad True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\
& \quad (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \\
& \hspace{15em} (62)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow \\
& \quad True)) \\
& \hspace{15em} (63)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in \\
& \quad A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \\
& \hspace{15em} (64)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\
& \quad (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\
& \quad p\ V0t)))))) \\
& \hspace{15em} (65)
\end{aligned}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (66)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (67)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow & (\forall V0P \in 2.(\forall V1Q \in 2. \\ & (\forall V2x \in A_{.27a}.(\forall V3x_{.27} \in A_{.27a}.(\forall V4y \in A_{.27a}. \\ & (\forall V5y_{.27} \in A_{.27a}.(((p V0P) \Leftrightarrow (p V1Q)) \wedge ((p V1Q) \Rightarrow (V2x = V3x_{.27})) \wedge \\ & ((\neg(p V1Q)) \Rightarrow (V4y = V5y_{.27})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebool_{.2ECOND}} \ A_{.27a}) \\ & V0P) V2x) V4y) = (ap (ap (ap (c_{.2Ebool_{.2ECOND}} \ A_{.27a}) V1Q) V3x_{.27}) \\ & V5y_{.27})))))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow & ((\forall V0t1 \in A_{.27a}.(\forall V1t2 \in \\ & A_{.27a}.((ap (ap (ap (c_{.2Ebool_{.2ECOND}} \ A_{.27a}) \ c_{.2Ebool_{.2ET}}) V0t1) \\ & V1t2) = V0t1))) \wedge (\forall V2t1 \in A_{.27a}.(\forall V3t2 \in A_{.27a}.((ap \\ & (ap (ap (c_{.2Ebool_{.2ECOND}} \ A_{.27a}) \ c_{.2Ebool_{.2EF}}) V2t1) V3t2) = V3t2)))))) \end{aligned} \quad (69)$$

Assume the following.

$$\forall A_{.27a}.nonempty \ A_{.27a} \Rightarrow (\forall V0x \in A_{.27a}.((ap (c_{.2Ecombin_{.2EI}} \ A_{.27a}) V0x) = V0x)) \quad (70)$$

Theorem 1

$$\begin{aligned} \forall A_{.27t}.nonempty \ A_{.27t} \Rightarrow & \forall A_{.27w}.nonempty \ A_{.27w} \Rightarrow (\\ & \forall V0b \in 2.(\forall V1y \in (ty_{.2Ebinary_{.ieee_{.2Efloat}}} \ A_{.27t} \\ & A_{.27w}).(\forall V2x \in ty_{.2Erealax_{.2Ereal}}.((p (ap (ap \ c_{.2Erealax_{.2Ereal_{.lt}}} \\ & V2x) (ap \ c_{.2Erealax_{.2Ereal_{.neg}}} (ap (c_{.2Ebinary_{.ieee_{.2Elargest}}} \\ & A_{.27t} \ A_{.27w}) (c_{.2Ebool_{.2Ethe_{.value}}} (ty_{.2Epair_{.2Eprod}} \ A_{.27t} \\ & A_{.27w})))))) \Rightarrow ((ap (ap (ap (c_{.2Ebinary_{.ieee_{.2Efloat_{.round}}} \ A_{.27t} \\ & A_{.27w}) \ c_{.2Ebinary_{.ieee_{.2EroundTowardNegative}}} \ V0b) V2x) = (\\ & ap (c_{.2Ebinary_{.ieee_{.2Efloat_{.minus_{.infinity}}} \ A_{.27t} \ A_{.27w}}) \\ & (c_{.2Ebool_{.2Ethe_{.value}}} (ty_{.2Epair_{.2Eprod}} \ A_{.27t} \ A_{.27w})))))) \end{aligned}$$