

thm\_2Ebinary\_ieee\_2Efloat\_sub\_plus\_infinity\_finite  
(TMMwhGYdRY-  
Giz4AU n5DSkYji1sWMG8GJpFT)

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Let  $ty\_2Ebinary\_ieee\_2Eflags : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Eflags \quad (1)$$

Let  $c\_2Ebool\_2EARB : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow c\_2Ebool\_2EARB\ A.27a \in A.27a \quad (2)$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a})))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Ecombin\_2EK$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0x \in A.27a.(\lambda V1y \in A.27b.V0x))$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_AfterRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (3)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Underflow\_BeforeRounding\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (4)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Precision\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (5)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_Overflow\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (6)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (7)$$

Let  $c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd \in ((ty\_2Ebinary\_ieee\_2Eflags^{ty\_2Ebinary\_ieee\_2Eflags})^{(2^2)}) \quad (8)$$

**Definition 6** We define  $c\_2Ebinary\_ieee\_2Eclear\_flags$  to be  $(ap (ap c\_2Ebinary\_ieee\_2Eflags\_DivideByZero\_fupd))$

**Definition 7** We define  $c\_2Ebinary\_ieee\_2Einvalidop\_flags$  to be  $(ap (ap c\_2Ebinary\_ieee\_2Eflags\_InvalidOp\_fupd))$

Let  $ty\_2Ebinary\_ieee\_2Efp\_op : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efp\_op A0 A1) \quad (9)$$

Let  $ty\_2Ebinary\_ieee\_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty\_2Ebinary\_ieee\_2Efloat A0 A1) \quad (10)$$

Let  $ty\_2Ebinary\_ieee\_2Errounding : \iota$  be given. Assume the following.

$$nonempty ty\_2Ebinary\_ieee\_2Errounding \quad (11)$$

Let  $c\_2Ebinary\_ieee\_2EFP\_Sub : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_2Ebinary\_ieee\_2EFP\_Sub A\_27t A\_27w \in (((ty\_2Ebinary\_ieee\_2Efp\_op A\_27t A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat A\_27t A\_27w)})^{(ty\_2Errounding)}) \quad (12)$$

Let  $ty\_2Elist\_2Elist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Elist\_2Elist A0) \quad (13)$$

Let  $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \quad (14)$$

Let  $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \quad (15)$$

Let  $ty\_2EfcP\_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2EfcP\_2Ecart\ A0\ A1) \quad (16)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Significand : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Significand\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27t)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (17)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (18)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (19)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (20)$$

**Definition 8** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 9** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (21)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (22)$$

**Definition 10** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (23)$$

**Definition 11** We define  $c\_Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmetic$

**Definition 12** We define  $c\_Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Ebool\_2Eitself : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Ebool\_2Eitself A0) \quad (24)$$

Let  $c\_2Ebool\_2Ethe\_value : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Ebool\_2Ethe\_value A\_27a \in (ty\_2Ebool\_2Eitself A\_27a) \quad (25)$$

Let  $c\_2Efcf\_2Edimindex : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Efcf\_2Edimindex A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself A\_27a)}) \quad (26)$$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (27)$$

Let  $ty\_2Efcf\_2Efinite\_image : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Efcf\_2Efinite\_image A0) \quad (28)$$

**Definition 13** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 14** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_7E$

**Definition 15** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

**Definition 16** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 17** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 18** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 19** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C$

**Definition 20** We define  $c\_2Efcf\_2Efinite\_index$  to be  $\lambda A\_27a : \iota.(ap (c\_2Emin\_2E\_40 (A\_27a^{ty\_2Enum\_2Enum}$

Let  $c\_2Efcf\_2Edest\_cart : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Efcf\_2Edest\_cart A\_27a A\_27b \in ((A\_27a^{(ty\_2Efcf\_2Efinite\_image A\_27b)})^{(ty\_2Efcf\_2Ecart A\_27a A\_27b)}) \quad (29)$$

**Definition 21** We define  $c\_2Efcf\_2Efcf\_index$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in (ty\_2Efcf\_2Ecart A\_27a$

**Definition 22** We define  $c\_Ewords\_Eword\_msb$  to be  $\lambda A.27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A.27a).(ap$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (30)$$

**Definition 23** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (31)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (32)$$

Let  $c\_2Earithmetic\_2EEXP : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (33)$$

**Definition 24** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A.27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A.27a.(\lambda V2t2 \in A.27a.($

**Definition 25** We define  $c\_2Ebit\_2ESBIT$  to be  $\lambda V0b \in 2.\lambda V1n \in ty\_2Enum\_2Enum.(ap\ (ap\ (ap\ (c\_2Ebo$

Let  $c\_2Esum\_num\_2ESUM : \iota$  be given. Assume the following.

$$c\_2Esum\_num\_2ESUM \in ((ty\_2Enum\_2Enum^{(ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})})^{ty\_2Enum\_2Enum}) \quad (34)$$

**Definition 26** We define  $c\_2Ewords\_2Ew2n$  to be  $\lambda A.27a : \iota.\lambda V0w \in (ty\_2Efc\_2Ecart\ 2\ A.27a).(ap\ (ap\ c$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (35)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (36)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (37)$$

**Definition 27** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E.40\ (t$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (38)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (39)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}}) \quad (40)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 29** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (41)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 31** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (42)$$

**Definition 32** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Ewords\_2EINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EINT\_MAX\ A\_27a \in (ty\_2Eenum\_2Eenum)^{(ty\_2Ebool\_2Eitsel\ A\_27a)} \quad (43)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Exponent\ A\_27t\ A\_27w \in ((ty\_2EfcP\_2Ecart\ 2\ A\_27w)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (44)$$

Let  $ty\_2Eone\_2Eone : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eone\_2Eone \quad (45)$$

Let  $c\_2Ebinary\_ieee\_2Efloat\_Sign : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_Sign\ A\_27t\ A\_27w \in ((ty\_2Efc\_2Ecart\ 2\ ty\_2Eone\_2Eone)^{(ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)}) \quad (46)$$

Let  $c\_2Erealax\_2Etreax\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreax\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (47)$$

**Definition 33** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (48)$$

**Definition 34** We define  $c\_2Ebit\_2EDIV\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2EMOD : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (49)$$

**Definition 35** We define  $c\_2Ebit\_2EMOD\_2EXP$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 36** We define  $c\_2Ebit\_2EBITS$  to be  $\lambda V0h \in ty\_2Enum\_2Enum.\lambda V1l \in ty\_2Enum\_2Enum.\lambda V$

**Definition 37** We define  $c\_2Ebit\_2EBIT$  to be  $\lambda V0b \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.(ap$

**Definition 38** We define  $c\_2Efc\_2EFCP$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0g \in (A\_27a^{ty\_2Enum\_2Enum}).(ap$

**Definition 39** We define  $c\_2Ewords\_2En2w$  to be  $\lambda A\_27a : \iota.\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (c\_2Efc\_2EFCP$

**Definition 40** We define  $c\_2Ebinary\_ieee\_2Efloat\_to\_real$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary$

Let  $ty\_2Ebinary\_ieee\_2Efloat\_value : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ebinary\_ieee\_2Efloat\_value \quad (50)$$

Let  $c\_2Ebinary\_ieee\_2EFloat : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EFloat \in (ty\_2Ebinary\_ieee\_2Efloat\_value^{ty\_2Erealax\_2Ereal}) \quad (51)$$

Let  $c\_2Ebinary\_ieee\_2ENaN : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2ENaN \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (52)$$

Let  $c\_2Ebinary\_ieee\_2EInfinity : \iota$  be given. Assume the following.

$$c\_2Ebinary\_ieee\_2EInfinity \in ty\_2Ebinary\_ieee\_2Efloat\_value \quad (53)$$

Let  $c\_2Ewords\_2EUINT\_MAX : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewords\_2EUINT\_MAX\ A\_27a \in (ty\_2Enum\_2Enum^{(ty\_2Ebool\_2Eitself\ A\_27a)}) \quad (54)$$

**Definition 41** We define  $c\_Ewords\_Eword\_T$  to be  $\lambda A\_27a : \iota.(ap (c\_Ewords\_Een2w A\_27a) (ap (c\_Ew$

**Definition 42** We define  $c\_Ebinary\_ieee\_Efloat\_value$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_EBinary\_$

Let  $c\_EBinary\_ieee\_Efloat\_value\_CASE : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow c\_EBinary\_ieee\_Efloat\_value\_CASE \\ A\_27a \in & (((A\_27a^{A\_27a})^{A\_27a})^{(A\_27a^{ty\_Erealax\_Ereal})})^{ty\_EBinary\_ieee\_Efloat\_value}) \end{aligned} \quad (55)$$

**Definition 43** We define  $c\_EBinary\_ieee\_Efloat\_is\_nan$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_EBinary\_$

**Definition 44** We define  $c\_EBinary\_ieee\_Efloat\_is\_signalling$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_EBinary\_$

Let  $c\_Elist\_EEXISTS : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_Elist\_EEXISTS A\_27a \in ((2^{(ty\_Elist\_Elist A\_27a)})^{(2^{A\_27a})}) \quad (56)$$

**Definition 45** We define  $c\_EBinary\_ieee\_Echeck\_for\_signalling$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0l \in (ty\_EBinary\_$

**Definition 46** We define  $c\_Ewords\_Eword\_1comp$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_EfcP\_Ecart 2 A\_27a).$

Let  $c\_EBinary\_ieee\_Efloat\_Sign\_fupd : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty A\_27t \Rightarrow \forall A\_27w.nonempty A\_27w \Rightarrow c\_EBinary\_ieee\_ \\ A\_27t A\_27w \in & (((ty\_EBinary\_ieee\_Efloat A\_27t A\_27w)^{(ty\_EBinary\_ieee\_Efloat A\_27t A\_27w)})^{(ty\_EfcP\_Ecart 2 A\_27a)}) \end{aligned} \quad (57)$$

**Definition 47** We define  $c\_EBinary\_ieee\_Efloat\_negate$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_EBinary\_$

**Definition 48** We define  $c\_Ereal\_Ereal\_sub$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.$

Let  $c\_EBinary\_ieee\_ERoundTowardNegative : \iota$  be given. Assume the following.

$$c\_EBinary\_ieee\_ERoundTowardNegative \in ty\_EBinary\_ieee\_ERounding \quad (58)$$

Let  $c\_Erealax\_Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_lt \in ((2^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod ty\_Ehreal\_Ehreal)}) \quad (59)$$

**Definition 49** We define  $c\_Erealax\_Ereal\_lt$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal.$

**Definition 50** We define  $c\_Ereal\_Ereal\_lte$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.$

**Definition 51** We define  $c\_Ereal\_Eabs$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.(ap (ap (ap (c\_Ebool\_ECONV$



Let  $c\_2Ebinary\_ieee\_2Elargest : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Elargest \\ & A\_27t\ A\_27w \in (ty\_2Erealax\_2Ereal^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (60)$$

**Definition 52** We define  $c\_2Ebinary\_ieee\_2Efloat\_is\_finite$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27t\ A\_27w)$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ & A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (61)$$

**Definition 53** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2E\_2C\ x\ y))$

Let  $c\_2Epred\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_set\_2EGSPEC \\ & A\_27a\ A\_27b \in ((2^{A\_27a})^{(ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b}}) \end{aligned} \quad (62)$$

**Definition 54** We define  $c\_2Ebool\_2EIN$  to be  $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

**Definition 55** We define  $c\_2Ebinary\_ieee\_2Eis\_closest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s \in (2^{(ty\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27a\ A\_27b)})$

**Definition 56** We define  $c\_2Ebinary\_ieee\_2Eclosest\_such$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0p \in (2^{(ty\_2Ebinary\_ieee\_2Efloat\_is\_finite\ A\_27a\ A\_27b)})$

**Definition 57** We define  $c\_2Ebinary\_ieee\_2Eclosest$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(ap\ (c\_2Ebinary\_ieee\_2Eclosest\_such\ A\_27a\ A\_27b))$

Let  $c\_2Ebinary\_ieee\_2Efloat\_top : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_top \\ & A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (63)$$

**Definition 58** We define  $c\_2Ereal\_2Ereal\_gt$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.(V0x > V1y)$

Let  $c\_2Ebinary\_ieee\_2Efloat\_bottom : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_bottom \\ & A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (64)$$

**Definition 59** We define  $c\_2Ebool\_2ELET$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0f \in (A\_27b)^{A\_27a}).(\lambda V1x \in A\_27b.(V0f\ x))$

Let  $c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_ieee\_2Efloat\_minus\_infinity \\ & A\_27t\ A\_27w \in ((ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w))}) \end{aligned} \quad (65)$$

**Definition 60** We define  $c\_Ereal\_Ereal\_ge$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal$ .  
Let  $c\_Ebinary\_ieee\_Efloat\_plus\_infinity : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_plus\_infinity\ A\_27t\ A\_27w \in ((ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (66)$$

Let  $c\_Ebinary\_ieee\_Ethreshold : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Ethreshold\ A\_27t\ A\_27w \in (ty\_Erealax\_Ereal^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (67)$$

**Definition 61** We define  $c\_Ewords\_Eword\_lsb$  to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_Efcpcart\ 2\ A\_27a).(ap$

Let  $c\_Ebinary\_ieee\_Erounding2num : \iota$  be given. Assume the following.

$$c\_Ebinary\_ieee\_Erounding2num \in (ty\_Eenum\_Eenum^{ty\_Ebinary\_ieee\_Erounding}) \quad (68)$$

**Definition 62** We define  $c\_Ebinary\_ieee\_Erounding\_CASE$  to be  $\lambda A\_27a : \iota.\lambda V0x \in ty\_Ebinary\_ieee\_E$

**Definition 63** We define  $c\_Ebinary\_ieee\_Eround$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_Ebinary\_ie$

Let  $c\_Ebinary\_ieee\_Efloat\_plus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_plus\_zero\ A\_27t\ A\_27w \in ((ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (69)$$

Let  $c\_Ebinary\_ieee\_Efloat\_minus\_zero : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_Ebinary\_ieee\_Efloat\_minus\_zero\ A\_27t\ A\_27w \in ((ty\_Ebinary\_ieee\_Efloat\ A\_27t\ A\_27w)^{(ty\_Ebool\_Eitself\ (ty\_Epair\_Eprod\ A\_27t\ A\_27w))}) \quad (70)$$

**Definition 64** We define  $c\_Ebinary\_ieee\_Efloat\_is\_zero$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_Ebinar$

**Definition 65** We define  $c\_Ebinary\_ieee\_Efloat\_round$  to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_Ebinar$

Let  $c\_Ewords\_EINT\_MIN : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Ewords\_EINT\_MIN\ A\_27a \in (ty\_Eenum\_Eenum^{(ty\_Ebool\_Eitself\ A\_27a)}) \quad (71)$$

Let  $ty\_Esum\_Esum : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_Esum\_Esum\ A0\ A1) \quad (72)$$

Let  $c\_Ewords\_Edimword : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Ewords\_Edimword\ A\_27a \in (ty\_Eenum\_Eenum^{(ty\_Ebool\_Eitself\ A\_27a)}) \quad (73)$$

**Definition 66** We define `c_2Ewords_2Eword_2comp` to be  $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).$

**Definition 67** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 68** We define `c_2Ewords_2Enzcv` to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1b \in ($

Let `c_2Epair_2ESND` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (74)$$

Let `c_2Epair_2EFST` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (75)$$

**Definition 69** We define `c_2Epair_2EUNCURRY` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27$

**Definition 70** We define `c_2Ewords_2Eword_2ls` to be  $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2EfcP\_2Ecart\ 2\ A\_27a).\lambda V1b$

**Definition 71** We define `c_2Ebinary_2ieee_2Efloat_2is_2infinite` to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0x \in (ty\_2Ebin$

**Definition 72** We define `c_2Ebinary_2ieee_2Efloat_2round_2with_2flags` to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode$

**Definition 73** We define `c_2Ebinary_2ieee_2Efloat_2some_2qnan` to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0fp\_op \in (ty$

**Definition 74** We define `c_2Epair_2Epair_2CASE` to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0p \in (ty\_2Epair$

**Definition 75** We define `c_2Ebinary_2ieee_2Efloat_2sub` to be  $\lambda A\_27t : \iota.\lambda A\_27w : \iota.\lambda V0mode \in ty\_2Ebinary$

Let `c_2Ebinary_2ieee_2Efloat_2minus_2min` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_2ieee\_2Efloat\_2minus\_2min \\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_2ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)}) \end{aligned} \quad (76)$$

Let `c_2Ebinary_2ieee_2Efloat_2plus_2min` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow c\_2Ebinary\_2ieee\_2Efloat\_2plus\_2min \\ A\_27t\ A\_27w \in ((ty\_2Ebinary\_2ieee\_2Efloat\ A\_27t\ A\_27w)^{(ty\_2Ebool\_2Eitself\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)}) \end{aligned} \quad (77)$$

Assume the following.

$$\begin{aligned}
& \forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0a \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V1f \in (A\_27a^{ty\_2Erealax\_2Ereal}).(\forall V2v \in A\_27a. \\
& \quad (\forall V3v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\
& \quad A\_27a)\ (ap\ c\_2Ebinary\_ieee\_2Efloat\ V0a))\ V1f)\ V2v)\ V3v1) = (ap \\
& \quad V1f\ V0a)))))) \wedge ((\forall V4f \in (A\_27a^{ty\_2Erealax\_2Ereal}).(\forall V5v \in \\
& A\_27a.(\forall V6v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\
& \quad A\_27a)\ c\_2Ebinary\_ieee\_2EInfinity)\ V4f)\ V5v)\ V6v1) = V5v)))) \wedge \\
& \quad (\forall V7f \in (A\_27a^{ty\_2Erealax\_2Ereal}).(\forall V8v \in A\_27a. \\
& \quad (\forall V9v1 \in A\_27a.((ap\ (ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\_CASE \\
& \quad A\_27a)\ c\_2Ebinary\_ieee\_2ENaN)\ V7f)\ V8v)\ V9v1) = V9v1)))))) \\
& \hspace{15em} (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27t. \\
& nonempty\ A.27t \Rightarrow \forall A.27w.nonempty\ A.27w \Rightarrow (((ap\ (c.2Ebinary\_ieee.2Efloat\_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary\_ieee.2Efloat\_plus\_infinity \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe\_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = c.2Ebinary\_ieee.2EInfinity) \wedge (((ap\ (c.2Ebinary\_ieee.2Efloat\_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary\_ieee.2Efloat\_minus\_infinity \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe\_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = c.2Ebinary\_ieee.2EInfinity) \wedge ((\forall V0fp\_op \in \\
& \quad (ty.2Ebinary\_ieee.2Efp\_op\ A.27a\ A.27b).((ap\ (c.2Ebinary\_ieee.2Efloat\_value \\
& \quad A.27a\ A.27b)\ (ap\ (c.2Ebinary\_ieee.2Efloat\_some\_qnan\ A.27a \\
& \quad A.27b)\ V0fp\_op)) = c.2Ebinary\_ieee.2ENaN) \wedge (((ap\ (c.2Ebinary\_ieee.2Efloat\_value \\
& \quad A.27t\ A.27w)\ (ap\ (c.2Ebinary\_ieee.2Efloat\_plus\_zero\ A.27t \\
& \quad A.27w)\ (c.2Ebool.2Ethe\_value\ (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = \\
& \quad (ap\ c.2Ebinary\_ieee.2EFloat\ (ap\ c.2Ereal.2Ereal\_of\_num\ c.2Enum.2E0))) \wedge \\
& \quad (((ap\ (c.2Ebinary\_ieee.2Efloat\_value\ A.27t\ A.27w)\ (ap\ (c.2Ebinary\_ieee.2Efloat\_minus\_zero \\
& \quad A.27t\ A.27w)\ (c.2Ebool.2Ethe\_value\ (ty.2Epair.2Eprod\ A.27t \\
& \quad A.27w)))) = (ap\ c.2Ebinary\_ieee.2EFloat\ (ap\ c.2Ereal.2Ereal\_of\_num \\
& \quad c.2Enum.2E0))) \wedge (((ap\ (c.2Ebinary\_ieee.2Efloat\_value\ A.27t \\
& \quad A.27w)\ (ap\ (c.2Ebinary\_ieee.2Efloat\_plus\_min\ A.27t\ A.27w) \\
& \quad (c.2Ebool.2Ethe\_value\ (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = ( \\
& \quad ap\ c.2Ebinary\_ieee.2EFloat\ (ap\ (ap\ c.2Ereal.2E.2F\ (ap\ c.2Ereal.2Ereal\_of\_num \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\
& \quad (ap\ (ap\ c.2Ereal.2Epow\ (ap\ c.2Ereal.2Ereal\_of\_num\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (ap \\
& \quad c.2Earithmetic.2E.2B\ (ap\ (c.2Ewords.2EINT\_MAX\ A.27w)\ (c.2Ebool.2Ethe\_value \\
& \quad A.27w)))\ (ap\ (c.2Efc.2Edimindex\ A.27t)\ (c.2Ebool.2Ethe\_value \\
& \quad A.27t)))))) \wedge (((ap\ (c.2Ebinary\_ieee.2Efloat\_value\ A.27t\ A.27w) \\
& \quad (ap\ (c.2Ebinary\_ieee.2Efloat\_minus\_min\ A.27t\ A.27w)\ (c.2Ebool.2Ethe\_value \\
& \quad (ty.2Epair.2Eprod\ A.27t\ A.27w)))) = (ap\ c.2Ebinary\_ieee.2EFloat \\
& \quad (ap\ (ap\ c.2Ereal.2E.2F\ (ap\ c.2Ereal.2Ereal\_neg\ (ap\ c.2Ereal.2Ereal\_of\_num \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\
& \quad (ap\ (ap\ c.2Ereal.2Epow\ (ap\ c.2Ereal.2Ereal\_of\_num\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))))\ (ap\ (ap \\
& \quad c.2Earithmetic.2E.2B\ (ap\ (c.2Ewords.2EINT\_MAX\ A.27w)\ (c.2Ebool.2Ethe\_value \\
& \quad A.27w)))\ (ap\ (c.2Efc.2Edimindex\ A.27t)\ (c.2Ebool.2Ethe\_value \\
& \quad A.27t)))))))))
\end{aligned} \tag{79}$$

Assume the following.

$$True \tag{80}$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\
A.27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
& (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
& (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (82)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (( \\
& (p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (83)
\end{aligned}$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (84)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (85)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p \ V0t1) \Rightarrow \\
& ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (86)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\
& 2.(((p \ V0x) \Leftrightarrow (p \ V1x.27)) \wedge ((p \ V1x.27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y.27)))))) \Rightarrow \\
& (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x.27) \Rightarrow (p \ V3y.27)))))) \quad (87)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow ( \\
& \forall V0x \in A.27a.(\forall V1y \in A.27b.(\forall V2a \in A.27a.(\forall V3b \in \\
& A.27b.(((ap \ (ap \ (c.2Epair.2E.2C \ A.27a \ A.27b) \ V0x) \ V1y) = (ap \ (ap \\
& (c.2Epair.2E.2C \ A.27a \ A.27b) \ V2a) \ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \quad (88)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty \ A.27a \Rightarrow \forall A.27b.nonempty \ A.27b \Rightarrow \forall A.27c. \\
& nonempty \ A.27c \Rightarrow (\forall V0x \in A.27b.(\forall V1y \in A.27c.(\forall V2f \in \\
& ((A.27a^{A.27c})^{A.27b}).((ap \ (ap \ (c.2Epair.2Epair\_CASE \ A.27a \ A.27b \\
& A.27c) \ (ap \ (ap \ (c.2Epair.2E.2C \ A.27b \ A.27c) \ V0x) \ V1y)) \ V2f) = (ap \\
& (ap \ V2f \ V0x) \ V1y)))) \quad (89)
\end{aligned}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27t.nonempty\ A\_27t \Rightarrow \forall A\_27w.nonempty\ A\_27w \Rightarrow ( \\ & \quad \forall V0mode \in ty\_2Ebinary\_ieee\_2Erounding. (\forall V1x \in \\ & (ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w). (\forall V2r \in ty\_2Erealax\_2Ereal. \\ & \quad ((ap\ (c\_2Ebinary\_ieee\_2Efloat\_value\ A\_27t\ A\_27w)\ V1x) = (ap \\ & c\_2Ebinary\_ieee\_2Efloat\ V2r)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_sub \\ & \quad A\_27t\ A\_27w)\ V0mode)\ (ap\ (c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity \\ & \quad A\_27t\ A\_27w)\ (c\_2Ebool\_2Ethe\_value\ (ty\_2Epair\_2Eprod\ A\_27t \\ & \quad A\_27w))))\ V1x) = (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Ebinary\_ieee\_2Eflags \\ & (ty\_2Ebinary\_ieee\_2Efloat\ A\_27t\ A\_27w))\ c\_2Ebinary\_ieee\_2Eclear\_flags) \\ & \quad (ap\ (c\_2Ebinary\_ieee\_2Efloat\_plus\_infinity\ A\_27t\ A\_27w) \\ & \quad (c\_2Ebool\_2Ethe\_value\ (ty\_2Epair\_2Eprod\ A\_27t\ A\_27w)))))))))) \end{aligned}$$