

thm_2Ebinary_ieee_2Efloat__to__real__negate
 (TMcypmdnaX-
 owu1yPAbKKC7DfQYN8yqXenXP)

October 26, 2020

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Let $ty_2Ebinary_ieee_2Efloat : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Ebinary_ieee_2Efloat\ A0\ A1) \tag{3}$$

Let $ty_2EfcP_2Ecart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2EfcP_2Ecart\ A0\ A1) \tag{4}$$

Let $c_2Ebinary_ieee_2Efloat_Significand_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27u.nonempty\ A_27u \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand_fupd\ A_27t\ A_27u\ A_27w \in (((ty_2Ebinary_ieee_2Efloat\ A_27u\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \tag{5}$$

Let $c_2Ebinary_ieee_2Efloat_Exponent_fupd : \iota \Rightarrow \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow \forall A_27x.nonempty\ A_27x \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent_fupd\ A_27t\ A_27w\ A_27x \in (((ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27x)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)})^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \tag{6}$$

Let $ty_2Ebool_2Eitself : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ebool_2Eitself\ A0) \quad (7)$$

Let $c_2Efc_2Edimindex : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Efc_2Edimindex\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (8)$$

Let $c_2Ebinary_ieee_2Efloat_Significand : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Significand\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27t)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (9)$$

Let $c_2Ewords_2EINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ewords_2EINT_MAX\ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself\ A_27a)}) \quad (10)$$

Let $c_2Ebinary_ieee_2Efloat_Exponent : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_2Ebinary_ieee_2Efloat_Exponent\ A_27t\ A_27w \in ((ty_2Efc_2Ecart\ 2\ A_27w)^{(ty_2Ebinary_ieee_2Efloat\ A_27t\ A_27w)}) \quad (11)$$

Let $c_2Ebool_2Ethe_value : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2Ethe_value\ A_27a \in (ty_2Ebool_2Eitself\ A_27a) \quad (12)$$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (13)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (14)$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o\ (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 3 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (16)$$

Definition 4 We define $c_Ebool_2E_2T$ to be $(ap (ap (c_Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 5 We define $c_Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_Emin_2E_3D (2^{A-27a})) (\lambda V1x \in 2.V1x)) (\lambda V2x \in 2.V2x)))$

Definition 6 We define c_Eenum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap c_Eenum_2EABS_num (ap (ap (c_Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (17)$$

Definition 7 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B (ap (ap (c_Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))))$

Definition 8 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \quad (18)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealx_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealx_2Ereal}) \quad (20)$$

Let $ty_2Efcpx_2Efinite_image : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Efcpx_2Efinite_image\ A0) \quad (21)$$

Definition 9 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 10 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p \Rightarrow q)$ of type ι .

Definition 11 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E_21))$

Definition 12 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Definition 13 We define $c_Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A)\ \lambda P)$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c_Emin_2E_40\ A_27a))))$

Definition 15 We define $c_Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_Ebool_2E_3F_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap c_Ebool_2E_2F_5C (ap (ap (c_Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))))))$

Definition 17 We define $c_Efcpx_2Efinite_index$ to be $\lambda A_27a : \iota.(ap (c_Emin_2E_40\ (A_27a^{ty_2Enum_2Enum})))$

Let $c_2EfcP_2Edest_cart : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2EfcP_2Edest_cart \\ A_27a\ A_27b \in ((A_27a^{(ty_2EfcP_2Efinite_image\ A_27b)})^{(ty_2EfcP_2Ecart\ A_27a\ A_27b)}) \end{aligned} \quad (22)$$

Definition 18 We define $c_2EfcP_2EfcP_index$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in (ty_2EfcP_2Ecart\ A_27a\ A_27b)$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (23)$$

Definition 19 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 20 We define c_2Ebit_2ESBIT to be $\lambda V0b \in 2.\lambda V1n \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Esum_num_2ESUM : \iota$ be given. Assume the following.

$$c_2Esum_num_2ESUM \in ((ty_2Enum_2Enum^{(ty_2Enum_2Enum^{ty_2Enum_2Enum})})^{ty_2Enum_2Enum}) \quad (24)$$

Definition 21 We define $c_2Ewords_2Ew2n$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2EfcP_2Ecart\ 2\ A_27a).(ap\ (ap\ (ap\ (c_2Ebit_2ESBIT$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (25)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (26)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal_REP_CLASS}) \quad (27)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E40\ (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Erealax_2Ereal_inv}) \quad (28)$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Etreal_eq}) \quad (29)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \quad (30)$$

Definition 23 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)$

Definition 24 We define $c_Erealax_Einv$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_ABS)$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)))(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) \quad (31)$$

Definition 25 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 26 We define c_Ereal_E2F to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Definition 27 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_Eenum_Eenum.(ap\ (ap\ c_Earithmic_EBIT1))$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)))(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) \quad (32)$$

Definition 28 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Let $ty_Eone_Eone : \iota$ be given. Assume the following.

$$nonempty\ ty_Eone_Eone \quad (33)$$

Let $c_Ebinary_ieee_Efloat_Sign : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\ 27t.\ nonempty\ A\ 27t \Rightarrow \forall A\ 27w.\ nonempty\ A\ 27w \Rightarrow c_Ebinary_ieee_Efloat_Sign\ A\ 27t\ A\ 27w \in ((ty_Efcpl_Ecart\ 2\ ty_Eone_Eone)(ty_Ebinary_ieee_Efloat\ A\ 27t\ A\ 27w)) \quad (34)$$

Let $c_Erealax_Etrealmul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealmul \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)) \quad (35)$$

Definition 29 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_neg)$

Definition 30 We define $c_Ebit_EDIV_EXP$ to be $\lambda V0x \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_Eenum.$

Let $c_Earithmic_E2D : \iota$ be given. Assume the following.

$$c_Earithmic_E2D \in ((ty_Eenum_Eenum\ ty_Eenum_Eenum\ ty_Eenum_Eenum)ty_Eenum_Eenum) \quad (36)$$

Let $c_Earithmic_EMOD : \iota$ be given. Assume the following.

$$c_Earithmic_EMOD \in ((ty_Eenum_Eenum\ ty_Eenum_Eenum\ ty_Eenum_Eenum)ty_Eenum_Eenum) \quad (37)$$

Definition 31 We define $c_Ebit_EMOD_EXP$ to be $\lambda V0x \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$.

Definition 32 We define c_Ebit_EBITS to be $\lambda V0h \in ty_Enum_Enum.\lambda V1l \in ty_Enum_Enum.\lambda V$

Definition 33 We define c_Ebit_EBIT to be $\lambda V0b \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum.(ap$

Definition 34 We define c_Efc_EFCP to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0g \in (A_27a^{ty_Enum_Enum}).(ap$

Definition 35 We define c_Ewords_En2w to be $\lambda A_27a : \iota.\lambda V0n \in ty_Enum_Enum.(ap (c_Efc_EFC$

Definition 36 We define $c_Ebinary_ieee_Efloat_to_real$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebinary$

Let $c_Ebinary_ieee_Efloat_Sign_fupd : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27t.nonempty\ A_27t \Rightarrow \forall A_27w.nonempty\ A_27w \Rightarrow c_Ebinary_ieee \\ & A_27t\ A_27w \in (((ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)^{(ty_Ebinary_ieee_Efloat\ A_27t\ A_27w)})^{(ty_Efc \\ & \hspace{15em} (38) \end{aligned}$$

Definition 37 We define $c_Ewords_Eword_1comp$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_Efc_Ecart\ 2\ A_27a).$

Definition 38 We define $c_Ecombin_EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x)$

Definition 39 We define $c_Ebinary_ieee_Efloat_negate$ to be $\lambda A_27t : \iota.\lambda A_27w : \iota.\lambda V0x \in (ty_Ebinary$

Definition 40 We define c_Ebool_ELET to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0f \in (A_27b^{A_27a}).(\lambda V1x \in A_27$

Definition 41 We define $c_Ecombin_ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27$

Definition 42 We define $c_Ecombin_EI$ to be $\lambda A_27a : \iota.(ap (ap (c_Ecombin_ES\ A_27a\ (A_27a^{A_27a})\ A$

Definition 43 We define $c_Earithmic_E_3E$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 44 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21\ 2)\ (\lambda V2t \in$

Definition 45 We define $c_Earithmic_E_3E_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 46 We define $c_Earithmic_E_3C_3D$ to be $\lambda V0m \in ty_Enum_Enum.\lambda V1n \in ty_Enum_Enum$

Definition 47 We define $c_Eprim_rec_EPRE$ to be $\lambda V0m \in ty_Enum_Enum.(ap (ap (ap (c_Ebool_E$

Let $c_Earithmic_E_2A : \iota$ be given. Assume the following.

$$\begin{aligned} & c_Earithmic_E_2A \in ((ty_Enum_Enum)^{ty_Enum_Enum})^{ty_Enum_Enum} \\ & \hspace{15em} (39) \end{aligned}$$

Definition 48 We define $c_Eenumer_EiZ$ to be $\lambda V0x \in ty_Enum_Enum.V0x.$

Definition 49 We define $c_Eenumer_EiDUB$ to be $\lambda V0x \in ty_Enum_Enum.(ap (ap c_Earithmic_E$

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2) \quad (40)$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (41)$$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal \ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod \ ty_2Ehreal_2Ehreal)}) \quad (42)$$

Definition 50 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 51 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \quad (43)$$

Let $c_2Ewords_2Edimword : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow c_2Ewords_2Edimword \ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself \ A_27a)}) \quad (44)$$

Definition 52 We define $c_2Ewords_2Eword_comp$ to be $\lambda A_27a : \iota. \lambda V0w \in (ty_2EfcP_2Ecart \ 2 \ A_27a).$

Let $c_2Ewords_2EUINT_MAX : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow c_2Ewords_2EUINT_MAX \ A_27a \in (ty_2Enum_2Enum^{(ty_2Ebool_2Eitself \ A_27a)}) \quad (45)$$

Definition 53 We define $c_2Ewords_2Eword_T$ to be $\lambda A_27a : \iota. (ap \ (c_2Ewords_2En2w \ A_27a) \ (ap \ (c_2Ew$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (((ap \ (ap \ c_2Earithmetic_2E_2D \ c_2Enum_2E0) \ V0m) = c_2Enum_2E0) \wedge ((ap \ (ap \ c_2Earithmetic_2E_2D \ V0m) \ c_2Enum_2E0) = V0m)))) \quad (46)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1k \in ty_2Enum_2Enum. (\forall V2r \in ty_2Enum_2Enum. ((\exists V3q \in ty_2Enum_2Enum. (V1k = (ap \ (ap \ c_2Earithmetic_2E_2B \ (ap \ (ap \ c_2Earithmetic_2E_2A \ V3q) \ V0n)) \ V2r)) \wedge (p \ (ap \ (ap \ c_2Eprim_rec_2E_3C \ V2r) \ V0n)))) \Rightarrow (ap \ (ap \ c_2Earithmetic_2EMOD \ V1k) \ V0n) = V2r)))) \quad (47)$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow ((ap (ap c_2Earithmetic_2EMOD c_2Enum_2E0) \\
& V0n) = c_2Enum_2E0)))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& c_2Enum_2E0) V0n)) \Rightarrow (((ap (ap c_2Earithmetic_2EDIV V0n) V0n) = \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \wedge \\
& ((ap (ap c_2Earithmetic_2EMOD V0n) V0n) = c_2Enum_2E0))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1y \in ty_2Enum_2Enum. (\\
& (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) V1y)) \Rightarrow (((ap (ap c_2Earithmetic_2EMOD \\
& V0x) V1y) = V0x) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0x) V1y))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2EEXP \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) \\
& V0n) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO))) \wedge ((ap (ap c_2Earithmetic_2EEXP V0n) \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = \\
& V0n)))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& \forall A_{.27t}.nonempty\ A_{.27t} \Rightarrow \forall A_{.27u}.nonempty\ A_{.27u} \Rightarrow \forall A_{.27w}. \\
& \quad nonempty\ A_{.27w} \Rightarrow \forall A_{.27x}.nonempty\ A_{.27x} \Rightarrow ((\forall V0f0 \in \\
& \quad ((ty_2EfcP_2Ecart\ 2\ A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}).(\forall V1f \in \\
& \quad (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27x})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd \\
& \quad A_{.27t}\ A_{.27w}\ A_{.27x})\ V0f0)\ V1f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V1f)))) \wedge ((\forall V2f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V3f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V2f0)\ V3f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V3f)))) \wedge ((\forall V4f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}. \\
& \quad (\forall V5f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V4f0)\ V5f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V5f)))) \wedge ((\forall V6f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V7f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand_fupd \\
& \quad A_{.27t}\ A_{.27u}\ A_{.27w})\ V6f0)\ V7f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V7f)))) \wedge ((\forall V8f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}. \\
& \quad (\forall V9f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Sign_fupd\ A_{.27t}\ A_{.27w})\ V8f0)\ V9f)) = \\
& \quad (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27w})\ V9f)))) \wedge \\
& \quad ((\forall V10f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}). \\
& \quad (\forall V11f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t}\ A_{.27x})\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_{.27t}\ A_{.27w}\ A_{.27x}) \\
& \quad V10f0)\ V11f)) = (ap\ (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27t} \\
& \quad A_{.27w})\ V11f)))) \wedge ((\forall V12f0 \in ((ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)^{(ty_2EfcP_2Ecart\ 2\ ty_2Eone_2Eone)}. \\
& \quad (\forall V13f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad (c_2EbinaRy_ieee_2Efloat_Sign\ A_{.27t}\ A_{.27w})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign_fupd \\
& \quad A_{.27t}\ A_{.27w})\ V12f0)\ V13f)) = (ap\ V12f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Sign \\
& \quad A_{.27t}\ A_{.27w})\ V13f)))) \wedge ((\forall V14f0 \in ((ty_2EfcP_2Ecart\ 2 \\
& \quad A_{.27x})^{(ty_2EfcP_2Ecart\ 2\ A_{.27w})}).(\forall V15f \in (ty_2EbinaRy_ieee_2Efloat \\
& \quad A_{.27t}\ A_{.27w}).((ap\ (c_2EbinaRy_ieee_2Efloat_Exponent\ A_{.27t} \\
& \quad A_{.27x})\ (ap\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent_fupd\ A_{.27t} \\
& \quad A_{.27w}\ A_{.27x})\ V14f0)\ V15f)) = (ap\ V14f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Exponent \\
& \quad A_{.27t}\ A_{.27w})\ V15f)))) \wedge ((\forall V16f0 \in ((ty_2EfcP_2Ecart\ 2\ A_{.27u})^{(ty_2EfcP_2Ecart\ 2\ A_{.27t})}). \\
& \quad (\forall V17f \in (ty_2EbinaRy_ieee_2Efloat\ A_{.27t}\ A_{.27w}).((ap\ (\\
& \quad (c_2EbinaRy_ieee_2Efloat_Significand\ A_{.27u}\ A_{.27w})\ (ap\ (ap\ (\\
& \quad c_2EbinaRy_ieee_2Efloat_Significand_fupd\ A_{.27t}\ A_{.27u}\ A_{.27w}) \\
& \quad V16f0)\ V17f)) = (ap\ V16f0\ (ap\ (c_2EbinaRy_ieee_2Efloat_Significand \\
& \quad A_{.27t}\ A_{.27w})\ V17f)))))))))))))
\end{aligned} \tag{52}$$

Assume the following.

$$True \quad (53)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (54)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee (\neg(p V0t)))) \quad (56)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1x \in A_27a.((ap (ap (c_2Ebool_2ELET A_27a A_27b) V0f) V1x) = (ap V0f V1x)))) \quad (57)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (58)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (59)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (60)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (61)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (62)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (63)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (64)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. (((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V0t1)\ V1t2) = V1t2)))) \quad (65)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (66)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \quad (67)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in 2. (\forall V2x \in A_27a. (\forall V3x_27 \in A_27a. (\forall V4y \in A_27a. (\forall V5y_27 \in A_27a. (((p\ V0P) \Leftrightarrow (p\ V1Q)) \wedge (((p\ V1Q) \Rightarrow (V2x = V3x_27)) \wedge ((\neg(p\ V1Q)) \Rightarrow (V4y = V5y_27)))))) \Rightarrow ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V0P)\ V2x)\ V4y) = (ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ V1Q)\ V3x_27)\ V5y_27)))))) \quad (68)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow ((\forall V0t1 \in A_27a. (\forall V1t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2ET)\ V0t1)\ V1t2) = V0t1))) \wedge (\forall V2t1 \in A_27a. (\forall V3t2 \in A_27a. ((ap\ (ap\ (ap\ (c_2Ebool_2ECOND\ A_27a)\ c_2Ebool_2EF)\ V2t1)\ V3t2) = V3t2)))) \quad (69)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap\ (ap\ (c_2Ecombin_2EK\ A_27a\ A_27b)\ V0x)\ V1y) = V0x))) \quad (70)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V \forall x \in A_{27a}. ((\text{ap } (c.2\text{Ecombin}_2\text{El } A_{27a}) V 0x) = V 0x)) \quad (71)$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c_2Earithmic_2EBIT1\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((c_2Earithmic_2EZERO = (ap\ c_2Earithmic_2EBIT2 \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = c_2Earithmic_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c_2Earithmic_2EBIT1\ V0n) = (ap\ c_2Earithmic_2EBIT1 \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c_2Earithmic_2EBIT2\ V0n) = (ap\ c_2Earithmic_2EBIT2 \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO)\ (ap\ c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Earithmic_2EZERO) \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V0n)\ c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT1\ V0n))\ (ap\ c_2Earithmic_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& (ap\ c_2Earithmic_2EBIT2\ V0n))\ (ap\ c_2Earithmic_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V1m))))))))) \\
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1b \in 2. (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3m \in ty_2Enum_2Enum. (((ap (ap (ap c_2Enumeral_2EiSUB \\
& V1b) c_2Earithmetic_2EZERO) V0x) = c_2Earithmetic_2EZERO) \wedge (\\
& ((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) c_2Earithmetic_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Enumeral_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT1 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) c_2Earithmetic_2EZERO) = (ap c_2Earithmetic_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmetic_2EBIT2 \\
& V2n)) (ap c_2Earithmetic_2EBIT2 V3m)) = (ap c_2Earithmetic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (75)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmetic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0)))) \\
& \hspace{15em} (76)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((p (ap c_2Earithmetic_2EVEN c_2Earithmetic_2EZERO)) \wedge \\
& \quad ((p (ap c_2Earithmetic_2EVEN (ap c_2Earithmetic_2EBIT2 V0n))) \wedge \\
& \quad ((\neg(p (ap c_2Earithmetic_2EVEN (ap c_2Earithmetic_2EBIT1 V0n)))) \wedge \\
& \quad \quad ((\neg(p (ap c_2Earithmetic_2EODD c_2Earithmetic_2EZERO))) \wedge ((\\
& \quad \quad \neg(p (ap c_2Earithmetic_2EODD (ap c_2Earithmetic_2EBIT2 V0n)))) \wedge \\
& \quad (p (ap c_2Earithmetic_2EODD (ap c_2Earithmetic_2EBIT1 V0n))))))))) \\
& \hspace{15em} (77)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad V1y) V0x)))) \\
& \hspace{15em} (78)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& \quad (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& \quad (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z)))))) \\
& \hspace{15em} (79)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Erealax_2Ereal_neg V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) \\
& \hspace{15em} (80)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x)))) \\
& \hspace{15em} (81)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x)) \\
& \hspace{15em} (82)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x)) \\
& \hspace{15em} (83)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Erealax_2Ereal_neg V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) \\
& \hspace{15em} (84)
\end{aligned}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x)) \quad (85)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg V0x)) (ap c_2Erealax_2Ereal_neg V1y)))))) \quad (86)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V0x)))) \Leftrightarrow (V0x = V1y)))) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg V0x)) V1y) = (ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)))))) \quad (90)$$

Assume the following.

$$(\forall V0y \in ty_2Erealax_2Ereal.(\forall V1x \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt V1x) V0y)) \Leftrightarrow (\neg (p (ap (ap c_2Ereal_2Ereal_lte V0y) V1x)))))) \quad (91)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal.((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) V1y)) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap (ap c_2Erealax_2Ereal_add V0x) V1y)))))) \quad (92)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (ap c_2Ereal_2Ereal_lte (ap c_2Erealax_2Ereal_neg V0x)) \\
& (ap c_2Erealax_2Ereal_neg V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V1y) V0x))))))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (ap c_2Ereal_2Ereal_lte V0x) (ap c_2Erealax_2Ereal_neg \\
& V1y))) \Leftrightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y)) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_add V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) (ap (ap c_2Erealax_2Ereal_mul \\
& V0x) V2z))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_add V0x) V1y)) V2z) = (ap (ap c_2Erealax_2Ereal_add \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V2z)) (ap (ap c_2Erealax_2Ereal_mul \\
& V1y) V2z))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& \quad (\forall V2a \in ty_2Enum_2Enum. (\forall V3y \in ty_2Erealax_2Ereal. \\
& \quad ((ap (ap c_2Ereal_2Epow V0x) c_2Enum_2E0) = (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad (((ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 V1n))) = \\
& \quad (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \wedge (((ap (ap c_2Ereal_2Epow \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V1n))) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)) \wedge (((ap (ap c_2Ereal_2Epow (ap c_2Ereal_2Ereal_of_num \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2a))) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V1n)) = (ap c_2Ereal_2Ereal_of_num (ap (ap c_2Earithmetic_2EEXP \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2a)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V1n)))))) \wedge (((ap (ap c_2Ereal_2Epow (ap c_2Erealax_2Ereal_neg \\
& \quad (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL V2a)))) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V1n)) = (ap (ap (ap (ap (c_2Ebool_2ECOND \\
& \quad (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}) (ap c_2Earithmetic_2EODD \\
& \quad (ap c_2Earithmetic_2ENUMERAL V1n))) c_2Erealax_2Ereal_neg \\
& \quad (\lambda V4x \in ty_2Erealax_2Ereal.V4x)) (ap c_2Ereal_2Ereal_of_num \\
& \quad (ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V2a)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V1n)))))) \wedge ((ap (ap c_2Ereal_2Epow \\
& \quad (ap (ap c_2Ereal_2E_2F V0x) V3y)) V1n) = (ap (ap c_2Ereal_2E_2F (\\
& \quad ap (ap c_2Ereal_2Epow V0x) V1n)) (ap (ap c_2Ereal_2Epow V3y) V1n))))))))))))) \\
& \hspace{15em} (98)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (99)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (100)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (101)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (102)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (103)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& (ap (c_2Ewords_2Edimword A_27a) (c_2Ebool_2Ethe_value A_27a))))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0n \in ty_2Enum_2Enum.(\\
& (ap (c_2Ewords_2Ew2n A_27a) (ap (c_2Ewords_2En2w A_27a) V0n)) = \\
& (ap (ap c_2Earithmetic_2EMOD V0n) (ap (c_2Ewords_2Edimword A_27a) \\
& (c_2Ebool_2Ethe_value A_27a))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0m \in ty_2Enum_2Enum.(\\
& \forall V1n \in ty_2Enum_2Enum. (((ap (c_2Ewords_2En2w A_27a) V0m) = \\
& (ap (c_2Ewords_2En2w A_27a) V1n)) \Leftrightarrow ((ap (ap c_2Earithmetic_2EMOD \\
& V0m) (ap (c_2Ewords_2Edimword A_27a) (c_2Ebool_2Ethe_value \\
& A_27a))) = (ap (ap c_2Earithmetic_2EMOD V1n) (ap (c_2Ewords_2Edimword \\
& A_27a) (c_2Ebool_2Ethe_value A_27a))))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0w \in (ty_2Efc_2Ecart \\
& 2 A_27a). (\exists V1n \in ty_2Enum_2Enum. ((V0w = (ap (c_2Ewords_2En2w \\
& A_27a) V1n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V1n) (ap (c_2Ewords_2Edimword \\
& A_27a) (c_2Ebool_2Ethe_value A_27a))))))
\end{aligned} \tag{109}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow ((ap (c_2Ewords_2Ew2n A_27a) (ap \\
& (c_2Ewords_2En2w A_27a) c_2Enum_2E0)) = c_2Enum_2E0)
\end{aligned} \tag{110}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow ((ap (c_2Ewords_2Ew2n A_27a) (ap \\
& (c_2Ewords_2En2w A_27a) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& c_2Earithmetic_2EZERO)))) = (ap c_2Earithmetic_2ENUMERAL (ap \\
& c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))
\end{aligned} \tag{111}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty.2Enum.2Enum. (\\
& \quad (ap\ (c.2Ewords.2Eword_1comp\ A.27a)\ (ap\ (c.2Ewords.2En2w\ A.27a)\ \\
& \quad V0n)) = (ap\ (c.2Ewords.2En2w\ A.27a)\ (ap\ (ap\ c.2Earithmetic.2E.2D \\
& \quad (ap\ (ap\ c.2Earithmetic.2E.2D\ (ap\ (c.2Ewords.2Edimword\ A.27a)\ \\
& \quad (c.2Ebool.2Ethe_value\ A.27a))))\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO))))\ (ap\ (ap \\
& \quad c.2Earithmetic.2EMOD\ V0n)\ (ap\ (c.2Ewords.2Edimword\ A.27a)\ (c.2Ebool.2Ethe_value \\
& \quad A.27a))))))
\end{aligned} \tag{112}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0n \in ty.2Enum.2Enum. (\\
& \quad (ap\ (c.2Ewords.2Eword_2comp\ A.27a)\ (ap\ (c.2Ewords.2En2w\ A.27a)\ \\
& \quad V0n)) = (ap\ (c.2Ewords.2En2w\ A.27a)\ (ap\ (ap\ c.2Earithmetic.2E.2D \\
& \quad (ap\ (c.2Ewords.2Edimword\ A.27a)\ (c.2Ebool.2Ethe_value\ A.27a)))) \\
& \quad (ap\ (ap\ c.2Earithmetic.2EMOD\ V0n)\ (ap\ (c.2Ewords.2Edimword\ A.27a)\ \\
& \quad (c.2Ebool.2Ethe_value\ A.27a))))))
\end{aligned} \tag{113}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0m \in ty.2Enum.2Enum. (\forall V1n \in ty.2Enum.2Enum. \\
& \quad ((p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V0m)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT1\ V1n)))) \Leftrightarrow ((V0m = (ap\ (ap\ c.2Earithmetic.2E.2D \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1\ V1n))) \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))) \vee \\
& \quad (p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V0m)\ (ap\ (ap\ c.2Earithmetic.2E.2D \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1\ V1n))) \\
& \quad (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))))) \wedge \\
& \quad (\forall V2m \in ty.2Enum.2Enum. (\forall V3n \in ty.2Enum.2Enum. (\\
& \quad (p\ (ap\ (ap\ c.2Eprim_rec.2E.3C\ V2m)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT2\ V3n)))) \Leftrightarrow ((V2m = (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT1\ V3n))) \vee (p\ (ap\ (ap\ c.2Eprim_rec.2E.3C \\
& \quad V2m)\ (ap\ c.2Earithmetic.2ENUMERAL\ (ap\ c.2Earithmetic.2EBIT1 \\
& \quad V3n)))))))))
\end{aligned} \tag{114}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow ((ap\ (c.2Ewords.2Eword_2comp \\
& \quad A.27a)\ (ap\ (c.2Ewords.2En2w\ A.27a)\ (ap\ c.2Earithmetic.2ENUMERAL \\
& \quad (ap\ c.2Earithmetic.2EBIT1\ c.2Earithmetic.2EZERO)))) = (c.2Ewords.2Eword_T \\
& \quad A.27a))
\end{aligned} \tag{115}$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow ((\text{ap } (c_2\text{Ewords_2Eword_1comp } A_{27a}) (\text{ap } (c_2\text{Ewords_2En2w } A_{27a}) c_2\text{Enum_2E0})) = (c_2\text{Ewords_2Eword_T } A_{27a})) \quad (116)$$

Assume the following.

$$((\text{ap } (c_2\text{Ewords_2Edimword } ty_2\text{Eone_2Eone}) (c_2\text{Ebool_2Ethe_value } ty_2\text{Eone_2Eone})) = (\text{ap } c_2\text{Earithmetic_2ENUMERAL } (\text{ap } c_2\text{Earithmetic_2EBIT2 } c_2\text{Earithmetic_2EZERO}))) \quad (117)$$

Theorem 1

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow (\forall V0x \in (ty_2\text{Ebinary_ieee_2Efloat } A_{27a} A_{27b}). ((\text{ap } (c_2\text{Ebinary_ieee_2Efloat_to_real } A_{27a} A_{27b}) (\text{ap } (c_2\text{Ebinary_ieee_2Efloat_negate } A_{27a} A_{27b}) V0x)) = (\text{ap } c_2\text{Erealax_2Ereal_neg } (\text{ap } (c_2\text{Ebinary_ieee_2Efloat_to_real } A_{27a} A_{27b}) V0x))))))$$